3

Microwave radiometers: functions, design concepts, characteristics

The purpose of this chapter is to consider the basic techniques and concepts of measuring thermal electromagnetic fluctuation radiation. Further, using the method of equivalent schemes, the basic notions of brightness and noise temperature are introduced, which have been widely used in the theory and practice of passive microwave remote observations. The functions and characteristics of components of passive remote sensing devices are considered in detail; and also the basic concepts of designing microwave radiometers and the methodology of measuring their basic frequency and power characteristics are discussed. The latter include: the fluctuation threshold sensitivity; the form of amplitude–frequency responses; and the radiometric and energy passbands of receiving systems. The question of the limiting sensitivity of a microwave radiometer is also considered.

3.1 BASIC TYPES OF PASSIVE MICROWAVE DEVICES

As we have already noted, the procedure of experimentally measuring the spectral density of a random signal of any physical nature, including a fluctuating electromagnetic field, will consist of three basic components:

- linear frequency filtering, whose parameters are specified by the physical features of the problem under study;
- nonlinear quadratic transformation of a signal;
- temporal accumulation of a signal intended for separating the constant component of a transformed signal, whose value is proportional to the power of an initial analysed signal.

Let us consider, first of all, the first element of a measurement procedure. The fact is that the physical space around us possesses a huge diversity of forms of thermal radiation spectra, beginning from almost ‘white’ noise up to quite

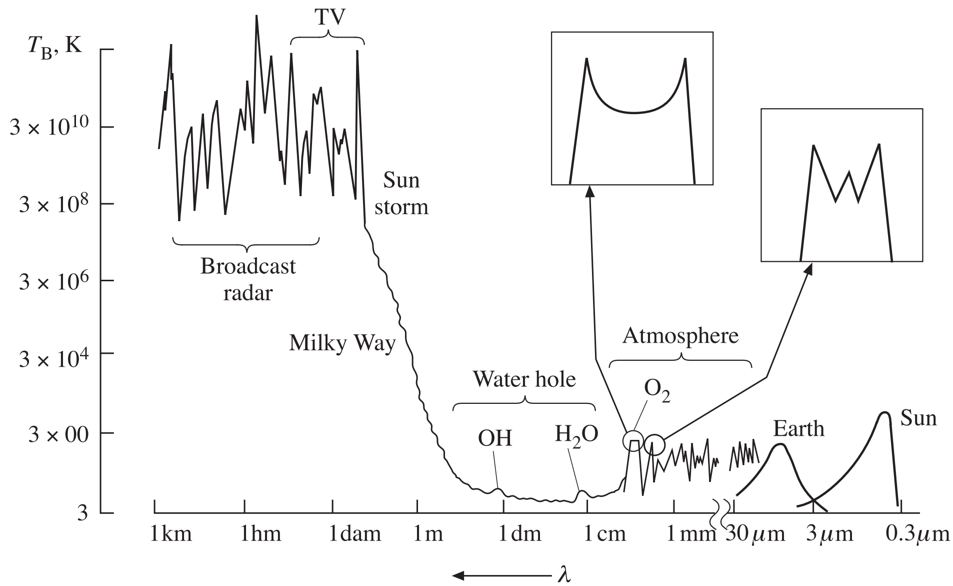

Figure 3.1. The qualitative picture (in temperature units) of the spectrum of electromagnetic emissions and artificial radiations.

complicated forms of spectra of radiation of gas media. Here, not only the intensity of a noise signal but also, most frequently, the form of a spectrum (or, as is sometimes said, the form of a line) bears useful physical information. To imagine a qualitative picture of natural thermal radiation, as well as the artificial electromagnetic radiations surrounding us (Figure 1.2), we shall turn to Figure 3.1, where electromagnetic radiation spectra are shown in the temperature brightness scale (see below) for a wide range of wavelengths, from the optical up to the kilometre band. If we move from optical frequencies into longer-wave regions, then here we should mention, first of all, the black-body radiation of the star nearest to the Earth, the Sun, whose radiation just ensures biological life on our planet. Then follows the nearly black-body radiation of our native planet, the Earth. And, further, almost the whole range of IR radiation up to the wavelength of 1 cm is 'occupied' by thermal radiation of the gases of the Earth's atmosphere in the form of a huge number of sharp line spectra (in essence, some paling of lines). The form of these lines is determined by the quantum character of electromagnetic energy absorption in gases and also depends on the ratio of contributions of vertical profiles of temperature and density of the corresponding gas to the radiation intensity. In this connection, the form of a particular line can be quite peculiar. So, the insertions in Figure 3.1 indicate the forms of strong absorption lines of atmospheric oxygen (they amount to about 100 individual lines in the range 55–65 GHz), which have merged into a powerful single line of 'two-finger' shape. The form of an oxygen line in the range of 118 GHz resembles the form of a tower of the Moscow Kremlin. All fine features of the forms of such lines play an important part in restoring the physical parameters of a gas medium and, consequently, they should not be 'lost' in the process of reception and processing of the original signal.

In cm and dm wavelength bands there exists some peculiar minimum of total radiation called the 'water hole' by radio-astronomers, because this band covers fairly weak (but very important in science) lines of absorption of water vapour (wavelength, 1.35 cm) and hydroxyl (OH) (wavelength, 18 cm). It is this band in which the ground radio-astronomical investigations have been carried out for a long time.

Beginning with the range of long decimetre wavelengths, the electromagnetic noise background sharply increases owing to the strong radio emission of the Sun and emission from our galaxy (the Milky Way). This emission has a strong spatial anisotropy and strong diurnal variations and, thus, highly impedes radiothermal investigations of the Earth's surface.

As we have noted above, the metre, decametre and longer-wave bands are saturated with a huge quantity of artificial emissions, which in the overwhelming majority of cases can be considered as quasi-chaotic with very complicated laws of distribution of amplitude fluctuations. Of course, the statistical methods considered in Chapter 2 can also be applied to these emissions: for example, the intensity of quasi-chaotic emission can be estimated. Estimations that have been carried out demonstrate striking results: the intensity of artificial sources of electromagnetic radiation is millions of times stronger (in comparable units) than solar radiation in the optical range. Artificial emissions are characterized by strong spatial-temporal variability, and it is virtually impossible to obtain a complete spectral picture of these emissions.

This qualitative review allows us to easily understand the principal importance of the procedure of primary filtering of the original signal for remote sensing tasks. In virtue of the huge diversity of the frequency forms of thermal emissions, the whole spectrum of natural electromagnetic emissions is usually subdivided into two large classes of emissions: the broadened continuum part of the continuous spectrum and the many-line absorption spectrum. Accordingly, radiometric receiving devices are subdivided from this point of view into two large classes: radiometric receiving devices with a fairly broad reception band (these devices are called continuous spectrum radiometers) and multichannel radiometer-spectrometers, designed for studying narrowband emissions (the lines of emissions). Certainly, this subdivision is fairly conventional, generally speaking. In recent times, radiometric systems of a combined type appeared which serve to study both the large-scale frequency features of a continuous spectrum and the emission lines 'built-in' in the continuous spectrum basement. An indicative example of such a system is the MTVZA radiometric complex intended for operation on the Russian 'Meteor-3M' spacecraft (see Chapter 14). On the other hand, multi-frequency continuous spectrum radiometers undoubtedly acquire the features typical for spectral devices. An example of such a system can serve the radiothermal instruments, the Advanced Microwave Scanning Radiometre (AMSR) installed for space flight aboard NASDA's ADEOS-2 and NASA's Aqua spacecraft in 2002 (see Chapter 14).

In accordance with the spectral-correlation approach, the concept of a radiometric receiving complex can be implemented in two forms: (a) as an analogue multichannel filter system (a filter analyser), and (b) as an autocorrelation receiver, which provides the formation of an autocorrelation function of a measured process with subsequent Fourier transformation into its frequency spectrum (Figure 3.2(a), (b)). The principal diagram of an analogue filter amplifying system and the position of filtering narrowband channels are presented in Figure 3.2(a), (c). Since in the overwhelming majority of cases in measurement practice the original external signal happens to be too weak to be subject to specialized processing, the signal required for solving a physical problem in the appropriate (working) frequency band undergoes considerable amplification by means of specialized broadband amplifiers. Further, multichannel narrowband filtering is performed in the working frequency band by means of a particular number of narrowband filters (Figure 3.2(c)). Their quantity and position in the working frequency band is determined both by the features of the physical problem, and by the tactical-technological requirements of the onboard measurement system. Whereas for restoring the atmospheric parameters in the troposphere, space radiothermal systems usually have from three to ten narrowband channels, for radio-astronomical ground-based systems the number of channels reaches 1024 and more. Then the quadratic transformation and low-pass filtering are performed in each channel. Further accumulation, recording, storage and representation of the final information are usually accomplished in specialized digital units on a computerized basis. In optics, similar types of multichannel devices have been called polychromators.

Autocorrelation reception is based on the decomposition of a time sequence of the basic signal according to the multichannel scheme. The growing time delay is introduced into each channel with subsequent multiplication of two signals, basic and delayed ones (Figure 3.2(b)) and accumulation of a signal (the time-averaging). The recording device forms the discrete values (corresponding to the number of channels) of the autocorrelation function and then transforms them into the spectral density of an original signal using the Fourier transformation. Further, the procedure of statistical evaluation of confidence intervals of the spectral density obtained is performed.

Both these approaches are equivalent, from the principal point of view. However, in remote sensing instrumentation for the microwave range, filter analysers of various types have prevailed for a long time for certain technological reasons. In recent times the scientific problems of modern radio-astronomy put on the agenda the production of broadband spectrum analysers with a high spectral resolution operating in real time. The manufacture of hybrid optical-digital processors, composed of optical-acoustic analysers and digital devices, promoted the solution of these problems. Owing to optimal distribution of processing functions between optical and digital parts, the optical-digital processor makes it possible to essentially increase the speed of processing and resolution capability. In such a type of hybrid processor, optical devices provide a high speed of integral transformations over the data set, and digital devices provide reliable and long-term storage and the required accuracy and flexibility of algorithms for subsequent data processing. The

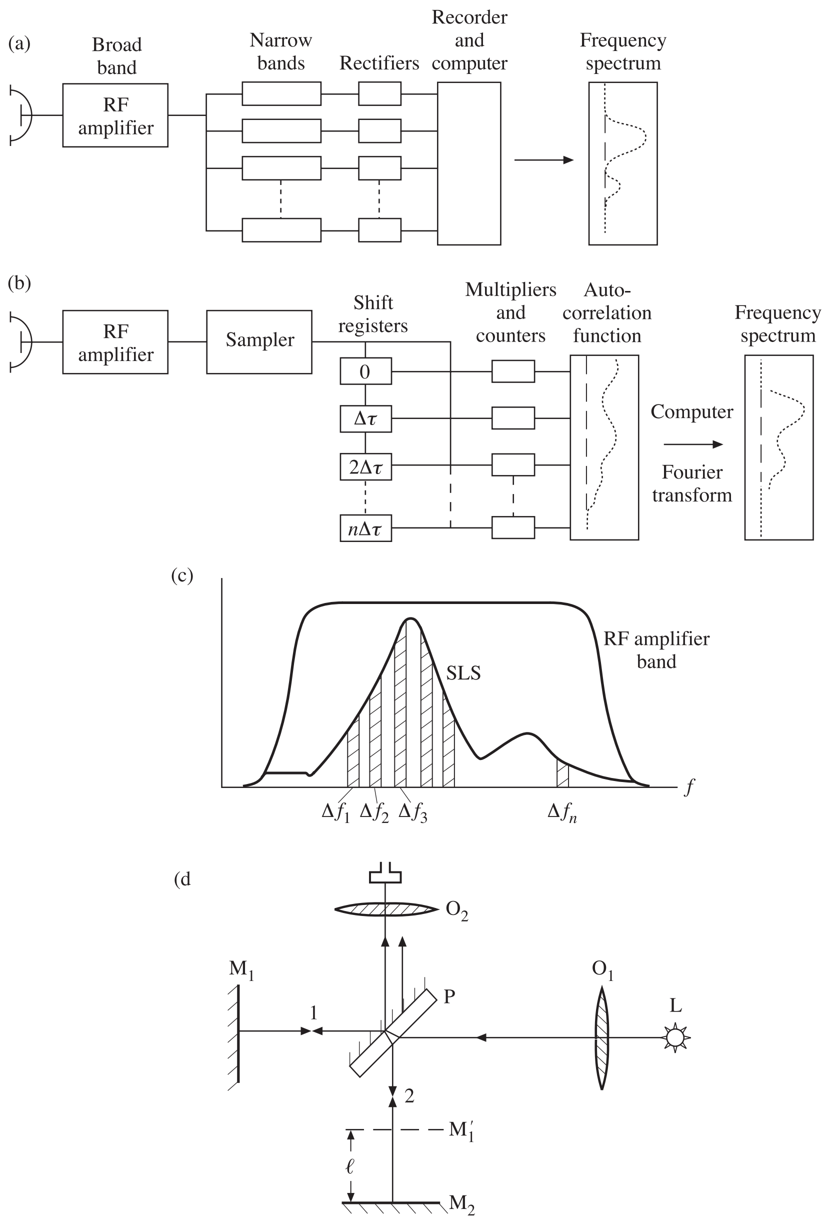

Figure 3.2. Microwave radiometers for spectral line observations: (a) an analogue filter bank RF-radio-frequencies system; (b) an autocorrelation spectrometer producing an autocorrelation function that can be Fourier-transformed to a frequency spectrum; (c) a schematic correlation between a spectral line shape (SLS) under study and narrowband filters; (d) the scheme of Michelson's interferometer. 1 and 2 are two coherent waves; L is a light source; O_1 and O_2 are convex lenses; P is a semitransparent plate; M_1 and M_2 are plane mirrors; M_1' is the imaginary reflection of M_1 mirror; 2l is a difference in optical path (correlation lag).

physical basis of the optical-acoustic Fourier processor rests upon the method of spatial separation of wavelengths by means of a dispersing element (the diffraction lattice or prism, for example), which is well known in optics. The multicomponent linear matrix photo-receiver is installed at the output plane of an optical processor. It performs the functions of a polychromator, an accumulator of a useful signal and a high-speed commutator. Some of the largest radio-telescopes are equipped now with optical-digital radiospectrometers, which have ensured considerable progress in the spectral measurement of radio-emission from both remote objects (galaxies) with large signal accumulation time, and the nearest star, the Sun, with a millisecond accumulation time (Esepkina *et al.*, 1997, 2000; Sorai *et al.*, 1998).

On the other hand, in the long-wavelength part of the submillimetre band receivers of the Fourier spectrometre type have been efficiently developed and utilized. These devices accomplish continuous coding of wavelengths with the help of interference modulation arising in the two-beam interferometer by changing of the optical difference of path (the simplest scheme of Michelson's interferometer is presented in Figure 3.2(d)). The receiver of the emission at the interferometer output provides the signal in time – the interferogram (or, in other words, the continuous autocorrelation function of the external process under investigation), which is subject to Fourier transformation on a computer or on a special processor for obtaining the required spectrum (Bell, 1972; Prochorov, 1984; Mandel and Wolf, 1995; Persky, 1995). Fourier spectrometers are most efficient for studying the spectra of rather weak sources (the Earth's atmosphere, the planets) in the IR and submillimetre ranges, as well as for the solution of super-high-resolution problems. The active advancement of Fourier spectroscopy methods into the longer-wave region indicates that in the very near future correlation receivers of the Fourier spectrometer type in the microwave range will actively compete with filter analysers.

It should be noted that, in the historical respect, early remote microwave systems were almost completely based on the use of a radio-astronomical practice, and have only gradually been developed in this independent direction.

3.2 BASIC COMPONENTS OF A PASSIVE MICROWAVE RADIOMETER AND THEIR FUNCTIONS

In the most general form, the scheme of studying physical objects by the use of passive radiothermal techniques can be presented as shown in Figure 3.3(a). The radiothermal complex represents a functionally combined antenna system, a microwave radiothermal receiver, recording and storage devices and a radio-engineering system for the transmission of obtained data to the information processing points. The radiothermal complex should possess the following properties:

- receive electromagnetic radiation in the particular frequency band from a particular spatial direction and at a particular body angle;

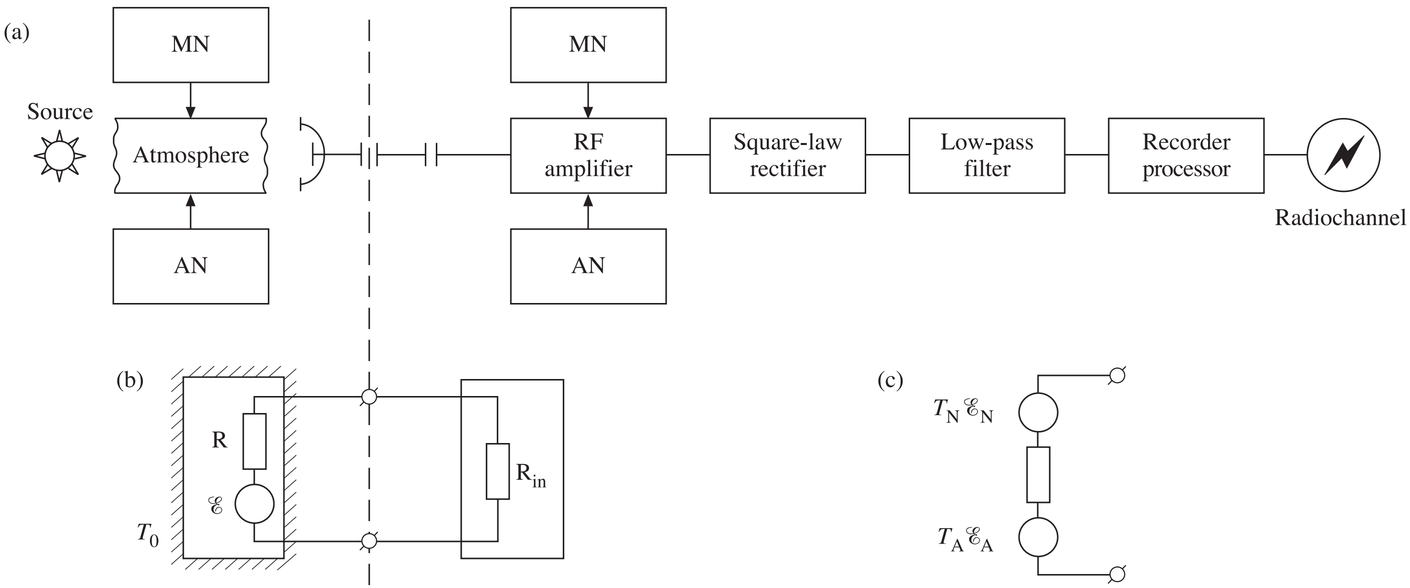

Figure 3.3. Schematic presentation of the transmission of information data from a natural object. (a) Physical scheme of the transmission of data: MN is a multiplicative noise; AN is an additive noise. (b) Equivalent scheme of the process of emission reception: E is equivalent noise source; R is equivalent active noise resistor; T_0 is equivalent temperature of the thermostat; R_{in} is the input resistance of the amplifier. (c) Equivalent scheme with due regard for the amplifier's noise. $\mathscr{E}_A$ and $\mathscr{E}_N$ are equivalent noise sources of the object under study and the amplifier; T_A is the antenna temperature; T_N is the noise temperature of the amplifier.

- possess a high sensitivity allowing it to reliably record variations in the thermal radiation of physical objects;
- provide the possibility of uniquely attributing a measured radiation signal to the spatial coordinates of corresponding emitting objects.

In this connection, the radiothermal complex designed for remote measurements should include at least four chief indispensable components:

(1) an antenna system required for providing observation of the surface under investigation and for transforming the electromagnetic wave from free space into a measured signal;
(2) a radiothermal receiver allowing it to record and measure the useful signal to the required accuracy;
(3) a preprocessing device providing: antenna system control, data acquisition, preprocessing, calibration and recording into a memory device;
(4) a device providing formation of the obtained information into the form required for transmission over communication links with subsequent thematic processing and mapping by means of a ground complex (the ground segment).

We will consider the functions of the first two components in more detail.

The antenna system is designed, primarily, for transforming electromagnetic waves, which propagate in free space, into the complicated modes of oscillations of electromagnetic waves propagating in guiding transmission lines (waveguides, coaxial cables). In free space the vector fields (see section 1.6) **E** and **H** are strictly transversal, i.e. both vectors are perpendicular to the propagation direction and, in addition, they are perpendicular to each other and form the right-hand orthogonal triple of vectors (of the TEM type). The ratio of amplitudes of electrical and magnetic fields in the planar wave (the wave resistance), propagating in free space, is strictly fixed and equals the value (in the SI system) of 376.6 ohms. However, in guiding systems the structure (or the mode) of propagating electromagnetic waves is different in principle: these waves can have both longitudinal and transversal components. The number of modes of oscillation can be infinite. The wave propagation characteristics (phase velocity, wavelength, wave resistance) in guiding systems depend on the ratio of the working wavelength and physical size of the guiding system. So, in coaxial cables the wave resistance usually equals fixed values of 50 or 75 ohms, whereas in waveguides the wave resistance can vary within wide limits – from 100 to 500 ohms depending on the type of a wave and geometrical size of a waveguide system. Thus, the antenna system should transform the oscillations of TEM type into the type of oscillation that propagates in the further antenna-feeder system and in the receiving complex directly. In this case, in accordance with the method of impedances, the antenna system should correlate the wave resistances of free space and of guiding systems (see below).

The second designation of antenna systems is the spatial-angular selection of a signal with the particular angular resolution required for solution of the stated problem, and for accomplishing spatial-angular scanning for the formation of the observation band (or observation frame). In addition, in each resolution element the antenna system should receive a signal with a particular polarization, i.e. with a particular vector character of presentation of intensities of electrical and magnetic fields with respect to the geometry of the object under investigation (to the planar surface, for example).

In virtue of prominent dispersion properties, the antenna system should possess fairly homogeneous frequency properties of its basic characteristics throughout the working frequency band.

The radiometric receiving device consists of a high-frequency amplifier, a quadratic device and a low-frequency filter. The functions and properties of the two latter functional elements have been considered above (see sections 2.7 and 2.8). The function of a high-frequency amplifier consists in amplification of the signal, received by the antenna, in the frequency band strictly determined by the physical problem, for subsequent quadratic transformation.

Note that when the useful signal passes through the atmosphere and through the amplifying device, external interferences can be of two types: additive noise and multiplicative noise (Figure 3.3(a)). The physical significance of their separation is fairly transparent. The first type of interference (or noise) is an independent process with respect to a useful signal (for example, the thermal radiation of the atmosphere or amplifier), and, thus, their intensities are added (which gives rise to the name of this type of interference). The second type of interference is related to the useful signal distortion by the atmospheric medium (or amplifier) through the effect on the amplitude and phase of a signal, and usually it is represented as a product of the useful signal value by the distorting factor (which gives rise to the name of this type of interference).

This separation is, certainly, fairly conventional; but, nevertheless, it is quite convenient in practice. We shall demonstrate this below on practical examples and shall repeatedly use such an approach hereafter.

3.3 THE LANGUAGE OF EQUIVALENT CIRCUITS: THE ANTENNA, RADIOBRIGHTNESS AND NOISE TEMPERATURES

In virtue of the huge diversity of technological implementations of antenna systems and high-frequency amplifiers in the microwave range, and to introduce physical uniformity in the measurement process, it was recognized as expedient to use simplified equivalent circuits based on the impedance method (see section 1.6). We shall present the whole input portion in the measurement process as some noise source of emf (electromotive force). This source will have its internal resistance (which will correspond to the energy of a signal received by the antenna system), and the input resistance of an amplifier, on which, in its turn, the power of an external signal, transmitted from the antenna system, should be given off (Figure 3.3(b)). The character of a signal received by the antenna system represents the Gaussian random process with a value of variance corresponding to the intensity of an external signal to be measured (see section 2.2). In accordance with the Nyquist theorem (see Chapter 4), the noise signal generated on the complex resistance $\dot{Z}(\mathrm{j}2\pi f)$ has a similar statistical structure; this signal forms the spectral density (over positive frequencies) of the noise signal $G^{+}(f)$ as follows:

$$G^{+}(f) = 4kT_0 \,\mathrm{Re}\,\dot{Z}(\mathrm{j}2\pi f), \tag{3.1}$$

where T_0 is the thermodynamic temperature of a thermostat, where the complex resistance is placed, and k is the Boltzmann constant (see Appendix A, Table A.4). If we represent the complex resistance as a purely active resistance (R) and take the investigated frequency band as a band-pass filter with band Δf, then the noise signal variance, or the mean square of noise emf, $\overline{\mathscr{E}}^2$, will be

$$\sigma^2 = \overline{\mathscr{E}}^2 = 4kT_0 R\,\Delta f. \tag{3.2}$$

Since the input resistance of the receiver represents a load resistance of the antenna, where the useful signal is directed, it is important to determine the conditions under which the maximum power from a source (the emf plays its part in this case) can be transmitted into the receiving device. This procedure is called the match (the conformity). Now we can easily obtain the conditions imposed on the values of resistances of the source (R) and of the input resistance of an amplifier (R_{in}), for obtaining the value of a maximum power given off on the input resistance. For this purpose we shall write the expression for power, P, given off on the input resistance, with allowance for the Joule–Lenz law, as follows:

$$P = I^2 R_{\mathrm{in}} = \frac{\mathscr{E}^2}{(R + R_{\mathrm{in}})^2} R_{\mathrm{in}}. \tag{3.3}$$

By equating the derivative of this expression with respect to R to zero, we can get the important relationship, namely, $R = R_{\mathrm{in}}$. In other words, for providing the maximum power transmission to the receiver input, it is necessary to fulfil the conditions (match conditions) of equality of the source resistance and the input resistance of the amplifier. In this case the time-averaged value of maximum power, $P_{\max}$, given off on the input resistance, will be equal, with allowance for the Nyquist formula, to

$$P_{\max} = \frac{\overline{\mathscr{E}}^2}{4R} = k\,T_0\,\Delta f. \tag{3.4}$$

The expression obtained is very important methodologically, since it stipulates the expediency of the introduction of the temperature system of units for characterizing the spectral density of a noise power given off on a matched load of the antenna at reception of the external noise electromagnetic radiation. For this purpose the antenna temperature is introduced as the equivalent thermodynamic temperature of a noise resistance, which is equal to the input resistance of an amplifier and whose spectral power density is equal to the spectral power of the received external signal. Thus, by equating the intensity (the value of a variance) of the external noise signal to the intensity of the introduced artificial source, we obtain the antenna temperature value (in absolute degrees) expressed in terms of the spectral density of the received external source $G(s)$ as

$$T_{\mathrm{A}}(f) = \frac{G(f)}{k}. \tag{3.5}$$

If the antenna system operates in such a mode, where the spectral density of the received signal corresponds to the spectral density of the physical emitting object itself, then in such a case the temperature of an artificial source is called brightness (or radiobrightness) temperature. We shall postpone the analysis of the relationship between brightness and antenna temperatures in the general form till Chapter 5. And now we note the following important point. Any amplifying instrument, being a physical object at a particular thermodynamic temperature, possesses its natural (thermal) fluctuation electromagnetic radiation in exactly the same frequency band where the amplification of an external signal takes place. Since these two sources (external and internal) are statistically independent, their interaction in the power sense can be reduced to the sum of variances at the amplifier's output. However, in practical and experimental respects it is much more convenient to consider their relationship normalized to the amplifier's input, taking into account the linear gain of a system. Thus, we shall have at the amplifying system's input as it were two statistically independent sources of a noise signal – from the external object under study and from the internal noises of an amplifier. The temperature of the latter source is naturally called the noise temperature of an amplifying system. The total temperature at the amplifying system's input will be equal in this case to the sum of the antenna temperature, caused by the energy from the source under study, and the noise temperature of an amplifier (Figure 3.3(c)). Certainly, the values of these temperatures can differ considerably. So, for studying important wave effects on the sea surface, reliable recording of radiothermal signals in the range from 0.1 to 10–15 K is required, whereas the best amplifying systems have noise temperatures ranging from 300 to 1000 K. The amplifiers, used in everyday conditions (radio receivers, TV sets), have noise temperatures reaching some millions and hundred millions absolute degrees, and they are not used straightforwardly for studying thermal emissions.

In spite of the seemingly artificial character of the temperature ideology introduced in the measurement process, we shall uncover below a serious physical significance of such an approach (see Chapters 4 and 5). This is associated, first of all, with the peculiarities of black-body radiation in the microwave region of the electromagnetic spectrum.

3.4 COMPENSATORY SCHEME OF NOISE SIGNAL MEASUREMENT

As we have already noted, the perfect (and, we note, optimum) device for receiving noise-type signals of various physical natures, including electromagnetic emissions, is the system consisting of the ideal (noiseless) amplifier, a quadratic detector and an integrator (the low-pass filter), that forms signal accumulation. The present measuring scheme is accomplished in all electromagnetic spectrum bands, beginning from the optical and up to the microwave and lower-frequency bands. Certainly, each band, in accordance with the working wavelength values, possesses its peculiarities, both in radiation receivers and in the general circuitry of an entire device.

However, the presence of unremovable thermal noise radiation in the amplifying device, as well as the fluctuation variations of the gain, essentially changes the basic measuring scheme presented above. This can be demonstrated by the following example taken from recent radio-astronomical practice. The radiothermal signal value, recorded by the RT-22 radio telescope (of the Crimean astrophysical observatory) from the Crab Nebula was equal to 2.5 K, whereas the noise temperature of the receiving device was 300 K with a passband of 1 MHz. Remembering relationships (2.96) and (2.97), we have the signal at the radiometer output in the following form: the constant component is proportional to the sum of the noise temperature of a receiving device and received signal, as well as the residual (after transformation) noise with the variance value equal to

$$\sigma_{\mathrm{F}}^2 = (\bar{V})^2 \frac{1}{2\Delta f\,\tau}. \tag{3.6}$$

Here account is taken of the fact that the effective passband of a low-pass filter is $\Delta F = \frac{1}{4\tau}$ (see relation (2.72)), and by Δf is meant the total passband of a high-frequency channel. As we have mentioned above, it is convenient to reduce relation (3.6), by means of appropriate calibrations, to the input of the whole receiving system and to consider the relation between the signal determining the intensity of radiation under study and the residual noise component from the same radiation just at the receiving system input. In this case the root-mean-square variation of a noise component will be equal to

$$\sqrt{\sigma_{\mathrm{F}}^2} = (T_{\mathrm{N}} + T_{\mathrm{S}}) \frac{1}{\sqrt{2\Delta f\,\tau}}, \tag{3.7}$$

and, accordingly, the noise 'pick-to-pick' of the Gaussian noise will be $6\sqrt{\sigma_{\mathrm{F}}^2}$. By substituting the aforementioned values of the receiving system's parameters (for $\tau = 1$ sec) into (3.7), we obtain the root-mean-square variation value as 0.2 K and the 'pick-to-pick' value as 1.2 K. Figure 3.4(a) shows the time recording of an output signal at an output of the receiving system of a radio telescope, when the source under investigation passes through the antenna pattern $(T_{\mathrm{N}} + T_{\mathrm{S}})$, and the recording without the presence of a source (T_{N}). The complexity of the measurement situation is fairly clear: against the background of a great signal (300 K) we must reliably record a useful signal, which is essentially (more than 100–200 times) lower in amplitude. Therefore, we must have an even more sensitive (by an order of magnitude at least) measurement system (in continuous signal). In order to record the small addition to an output signal caused by a useful signal, it was suggested that we compensate for the continuous voltage caused by intrinsic noises of a receiving device, by means of a special source of constant voltage placed at the receiving system's output. Thus, we arrive at the so-called compensatory scheme of noise signal measurement shown in Figure 3.5(a). This is the simplest scheme for

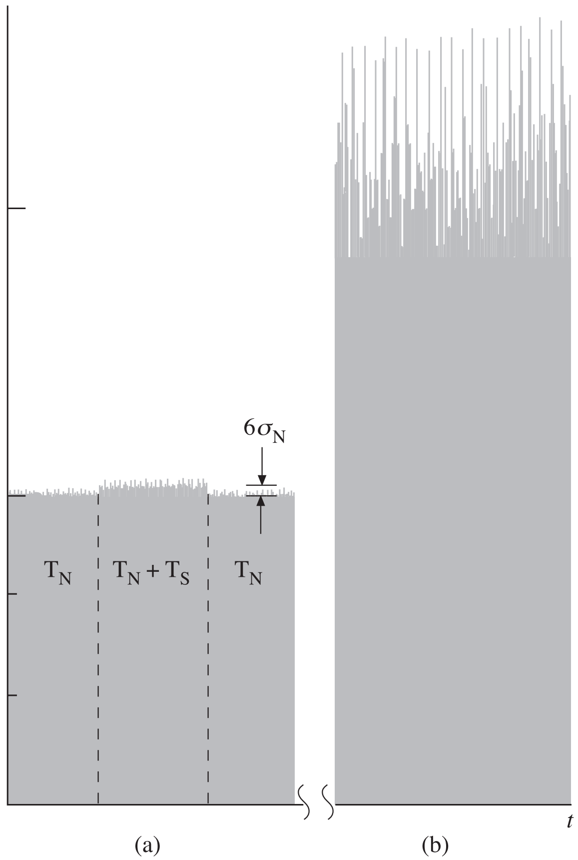

Figure 3.4. The output signal record of a radiometer (in arbitrary units). (a) Record with the square-law detector and the low-pass filter (the integrator); T_N and T_S are noise temperatures of the amplifier and the signal. (b) Record without the low-pass filter.

measuring the noise signal. However, its principal sensitivity is higher than in any other scheme. As an illustration, Figure 3.4(b) demonstrates the situation where the low-pass filter (the integrator) is absent from the system's output. Figure 3.4(b) visually indicates the advantages of using a low-pass filter that we have mentioned above many times (see section 2.8). The physical essentials of the compensatory scheme operation is as follows. As we have seen in (2.95) and (2.96) and in Figure 3.4(a), the output signal consists of a continuous component which is proportional to the intensity of a noise signal formed from the intrinsic noise of an amplifying system, and is equal to

$$\bar{V}_N = \beta\sigma_N^2 = \beta G_N(f)\,\Delta f = \beta T_N k\,\Delta f = k_1 G_A(f) T_N k\,\Delta f, \tag{3.8}$$

where $G_A(f)$ is the linear power gain of an amplifying system. Since this parameter determines the transition and amplification of the Wiener power spectrum of the

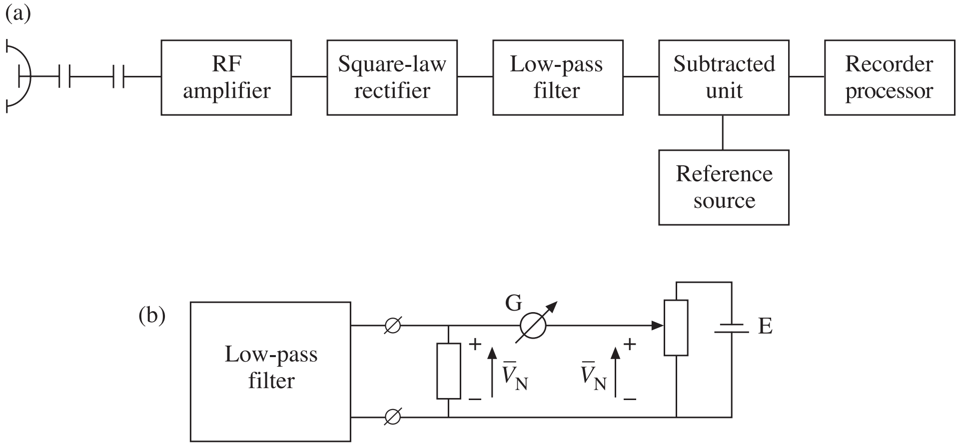

Figure 3.5. Functional scheme of a compensative radiometer. (a) Block diagram of a compensative radiometer. (b) Simplified variant of a subtracted circuit. G is a galvanometer; E is an electric potential source.

original signal, in experimental practice this function is called the frequency response of an amplifying system. It is equal to the square of the magnitude of the transmission coefficient of the high-frequency part of the receiver. Parameter k_1 is the linear transmission coefficient of the remaining components of an amplifying and recording system. The second constant (or slowly varying) component will be determined by the intensity of an external noise signal subject to measurement, and it is equal to

$$\bar{V}_S(t) = \beta\sigma_S^2 = k_1 G_A(f) T_S(t) k\,\Delta f. \tag{3.9}$$

The fluctuation component at the output will be mainly determined by an instrument's noise, and its RMS can be determined according to (3.7).

To compensate for the continuous component of an output signal caused by an amplifier's noise (3.8), the subtracting unit and the source of a compensatory (reference) constant signal are introduced into the circuit (see Figure 3.5(a)). The simplest version of such a device is presented in Figure 3.5(b); the compensation is accomplished by the voltage from the external electrical potential source. It is such compensatory devices which were the first instruments, historically, in producing the first compensatory radiometers. The compensatory voltage is set up for carrying out measurements of a particular type and does not change, usually, during the cycle of measurements. So, for example, such a scheme has been successfully used in radio-astronomical investigations of the Sun and other powerful radio-emission sources. It is important to note here that, in this case, not only the noises of a receiving device are compensated by relation (3.8), but also the signal caused by the mean value of the intensity of radio-emission of the source itself (for the Sun it equals 6000 K), is compensated by relation (3.9), and the radio-emission variations against the background of a powerful thermal emission of the source are investigated.

Under the conditions of onboard observations of highly spatially varying sources (Earth's radio-emission is an example of such a source), the compensatory techniques used above become rather inconvenient from the viewpoint of experimental implementation. However, the main factor limiting the use of compensatory devices is found to be the fluctuation mode of the system's gain variations (or, as we have said above, the multiplicative interferences). In many cases such interferences eliminate the advantages of compensatory devices. This situation will be described in more detail in the next section.

3.5 THE FLUCTUATION THRESHOLD SENSITIVITY OF RADIOMETRIC SYSTEMS

In this section we shall consider the most important characteristic of radiometric systems – their sensitivity. This characteristic has a lot of equivalent names: the power threshold of sensitivity, the threshold signal, the threshold sensitivity, the fluctuation threshold of sensitivity. A principal difficulty of introducing this characteristic lies in the fact that at the radiometric complex output we have quantities of different orders: a slowly varying signal from useful radiation subject to measurement and the fluctuation signal, being a 'remainder' of the noise component after quadratic transformation of both the main signal and the additive noise of an amplifier (see section 2.8 and equations (2.95), (2.96), (3.8) and (3.9)). On the basis of rich observational experience of the radio-astronomers and radio-physicists it was found worthwhile to introduce the following definition of the sensitivity of a radiometric receiver (Bunimovich, 1951; Troitskii, 1951; Esepkina *et al.*, 1973). By the threshold sensitivity of instruments for measuring the intensity of the fluctuation electromagnetic radiation is meant such a noise signal at the receiving system's input that is equal in magnitude, at the system's output, to the root-mean-square deviation of the fluctuation signal caused by the intrinsic noises of the amplifying channel.

To make the introduced definition clearer we consider the example of a signal recording at the output of a compensatory device (Figure 3.6). Since the mean value of a signal caused by instrument's noises is balanced, we shall have at the system's output only the fluctuation Gaussian random process caused by the residual transformed noises of an instrument with the root-mean-square deviation corresponding to relation (3.7). At time $t_0 - t_1$ the external noise signal of intensity $\Delta\delta$ arrives, which causes the deviation of a constant level at the system's output equal to one-sixth of the total noise 'track' and corresponding to the value of a root-mean-square deviation of the fluctuation component at the system's output. It is the signal of such intensity that represents the threshold sensitivity of a system. Now we shall turn to the quantitative side of the matter.

Let us remember the expression for the total intensity of a random signal at an output of the low-pass filter (see section 2.8 and equation (2.96)). However, it should be remembered that, in contrast to relation (2.96), the mean value of the signal (V_{F}) will be determined only by the external noise signal ΔT, and the variance of a fluctuation component (σ_{F}^2) will be determined both by the noises of an instrument

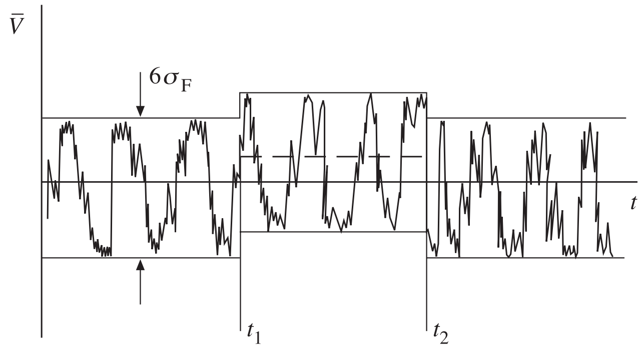

Figure 3.6. The output signal record (Gaussian random process) of a compensative radiometer. The input noise signal that is equal to the sensitivity threshold is given within $t_1 - t_2$. σ_F is RMS variation of noise signal after the low-pass filter (see equation (3.7)).

and by a signal (certainly, it is implied in this case, that the value $\Delta T \ll T_N$). Thus, equating $V_F = \sigma_F$, we have the following important relation for expressing the threshold sensitivity:

$$\Delta T = \sqrt{2} T_N \sqrt{\frac{\Delta F}{\Delta f}} = \frac{T_N}{\sqrt{2 \Delta f \, \tau}}. \tag{3.10}$$

If the time constant of a system equals 1 sec, then the threshold sensitivity is called normalized.

Now we pay attention to the fact that the introduction of the threshold sensitivity definition has been based, primarily, upon the practical experience of the experimenters, radio-astronomers, of separating a weak signal from the background of Gaussian fluctuations by means of visual averaging the time records. Subsequent studies in the field of recognition of signals and patterns have shown that the human eye – brain system represents a quite perfect instrument for separating very weak signals from the fluctuating background.

Now we shall estimate the normalized threshold sensitivity of a radiometric system for studying the continuous spectrum with the following characteristics: $T_N = 300$ K and a total passband of 10^9 Hz. It can easily be seen that the threshold sensitivity will be equal to 7×10^{-3} K, which is quite a high value. In this case it should be recognized that the receiving system can separate the useful noise signal from the background of the noise signal of the instrument having the same statistical characteristics and 4×10^4 times(!) exceeding the useful signal.

Special investigations in the field of sensitive measuring technique have shown that the compensatory scheme of random signal measurement is optimum and has, in principle, a higher threshold sensitivity than any other types of radiometric receivers. However, the further use of such a type of receiving devices has fairly quickly revealed a serious disadvantage of such schemes, associated with the unremovable fluctuations of the receiving system's gain. The physical sense of the noise arising can easily be understood by considering expression (3.9), where the constant component of a useful signal is proportional to the gain of a system. And, thus, it can be seen from this situation that fairly slow drifts of the gain value, i.e. $G_A(t)$ (or the multiplicative noises), will be non-separable from the variations of a useful signal $T_S(t)$. The occurrence of a fluctuating regime of multiplicative type in amplifying systems is a fully fundamental property and is associated with the physical properties of the instruments which comprise the amplifying system (electronic devices, solid-state instruments). This unremovable fluctuation regime, considered as a random process (or as an unremovable amplifier noise), possesses specific spectral density (the Wiener spectrum), namely, $G_{FN} \propto A/f^{\alpha}$ ($\alpha \approx 1$, and A is a constant depending on the type of amplifier) called the flicker-noise; and the physical effect itself is called the flicker-effect. A vast and scientific literature is devoted to radio-engineering and solid-state aspects of flicker-effect investigation, and so we shall not discuss this aspect here. Another issue is important for us: this type of fluctuation signal belongs to the so-called 'colour' noise (see section 2.5), and its contribution at very low frequencies can be very great because of the logarithmic divergence of the integral for calculating the noise variance (section 2.5). In other words, as the duration of the observation process increases (and, accordingly, the low-pass limit in the integral for calculating the variance decreases), the flicker-noise contribution will sharply grow and can essentially overlap the variations of a useful signal. The qualitative picture of the interrelation between frequency bands of flicker-noise, a low-pass filter, slow variations of a useful signal (V_S), and a low-pass limit of the variance integral (which is equal to the reverse value of the total experiment performing time) is presented in Figure 3.7. Depending on the type of amplifying device (constant A in the expression for the Wiener spectrum for flicker-noise), the detailed picture of the interrelation between the bands can greatly differ, of course. If the experiment performing time is limited, the flicker-noise variance σ^2_{FN} is also limited, generally speaking, i.e.

$$\sigma^2_{FN} = \int_F^{F_0} G_{FN}(f)\, df < M, \tag{3.11}$$

where $\Delta t = 1/F$ is the performing time, F is the low-pass limit of the variance integral and F_0 is the upper limit of the integral and the upper limit of a low-pass filter. Note, that the frequency range $[F_0, F]$ includes all frequency variations of the useful signal and flicker-noise which can be recorded by a radiometric instrument with the given characteristics. If the mentioned frequency range is sufficiently broad (the F_0/F ratio equals two or three orders of magnitude), then there exists some possibility of separating fairly rapid variations of the signal from the background of slow changes in the gain (for example, by additional frequency filtering). However, if the signal varies throughout the frequency range, then the separation of these components is virtually impossible.

For quantitative estimation of the effect of an amplifier's flicker-noise on the value of a receiving system's threshold sensitivity, we shall consider expression (3.9), where the relationship between the output signal after quadratic transformation and the system's gain value is expressed linearly. For this purpose we shall make use of the procedure of finding the correlation function at linear transformation of the

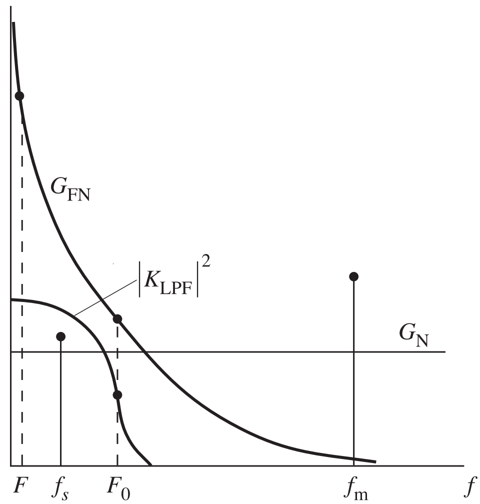

Figure 3.7. The correlation between output spectra of a compensative radiometer (in arbitrary units). G_{FN} is a flicker-noise spectrum; G_N is an amplifier noise spectrum as a result of square-law transformation for input noise signal; f_s is spectral signal component under study; $|K|^2$ is a frequency characteristic of the low-pass filter (the integrator); F and F_0 are frequencies of lower and upper bounds for the variance integral (equation (3.11)); f_m is the modulation frequency for a switched radiometer.

process (see section 2.6). So, for the value of signal variance after linear transformation (2.67) we have, with allowance for (3.8) and (3.11) and a subsequent low-pass filter,

$$B_{LF}^{FN}(0) = \sigma_{LF}^2 = \int_F^{F_0} G_{FN}(f)(k_1 T_N k \Delta f)^2 |\dot{K}(j2\pi f)|^2 \, df. \tag{3.12}$$

Assuming that in the filter band $|K(j2\pi f)|^2 = 1$, we obtain the estimate for a signal variance caused by the fluctuation regime of a gain (the flicker-noise):

$$\sigma_{LF}^2 \approx (k_1 k T_N \, \Delta f)^2 \, \sigma_{FN}^2. \tag{3.13}$$

Using the same methodical approach, as we applied above for determining the threshold sensitivity, we shall estimate the threshold sensitivity determined by the flicker-noise only. For this purpose we shall equate the value of the signal which is caused by the threshold signal at the system's input (3.9), to the value of root-mean-square deviation of a signal caused by the flicker-noise only (3.13). Thus, the threshold sensitivity value caused by the flicker-noise only, is equal to

$$\Delta T_{FN} = T_N \frac{\sqrt{\sigma_{FN}^2}}{G_A}. \tag{3.14}$$

Experiments have shown (Esepkina *et al.*, 1973), that in real amplifiers the flicker-noise RMS equals $\sigma_{FN} \approx (10^{-2} \div 10^{-3}) G_A$. Since the output signal fluctuations caused by intrinsic noises of an amplifier and the fluctuations caused by multiplicative variations of the gain can be considered as independent random processes, the expression for the sensitivity of a compensatory radiometer, with regard to amplification fluctuations, should be written as follows:

$$\Delta T = \sqrt{\Delta T_{\mathrm{N}}^2 + \Delta T_{\mathrm{N}}^2} = T_{\mathrm{N}}\sqrt{\frac{1}{2\Delta f\,\tau} + \left(\frac{\sigma_{\mathrm{FN}}}{G_{\mathrm{A}}}\right)^2}. \tag{3.15}$$

Substituting all the parameter values presented above, we can easily see that the threshold sensitivity equals $\sim$3 K, which means that the sensitivity is nearly 400 times lower than the ideal version of an instrument. Such a considerable loss in the equipment sensitivity requires consideration of completely new designs of observational instruments that would eliminate the influence of gain instabilities. The idea arises of some stable pilot-signal which would pass throughout the amplification channel together with the basic signal and, by analysing the change of a pilot-signal, it would be possible to monitor the drifting of the gain and to compensate for them quite quickly. Such a type of instrument, and the technique, were proposed by Professor R. Dicke in 1946. The measuring technique was called the modulation method of noise signal measurement, and the instrument itself was called the modulation radiometer, or the Dicke-type (or switched) radiometer.

3.6 THE MODULATION METHOD OF NOISE SIGNAL MEASUREMENT

The idea of a stable pilot-signal, which, along with the main (working) signal, passed through all transformation stages in a receiving device and, naturally, 'bore' in itself all 'parasitic' features of the amplification process with their consequent compensation, appeared so attractive in the experimental respect that it is now used in the vast majority of devices intended for recognizing a weak signal and separating it from the background of various parasitic variations in a receiver. So, this methodology is widely used at reception and recording of electromagnetic radiation in a wide wavelength range, beginning at the optical and up to super-low-frequency wavelength. This technique is widely used for the reception of acoustic signals and in mechanical systems (the recording of small oscillations or vibrations). The particular technological implementation of an instrument constructed according to this principle can be quite peculiar in some cases (for example, of hybrid type – the optical-mechanical devices) in IR engineering and Michelson interferometers (Bell, 1972; Persky, 1995)). And with such a complicated technological implementation it is not always possible to isolate directly all principal components of this technique. A fairly rich technical literature is devoted to the applied usage of modulation methods (or their improved modifications). Here we are interested in using this method in radiometric microwave receivers of weak electromagnetic emissions.

3.6.1 Modulation radiometer block-diagram

To implement the basic idea of the modulation method we select the main signal and the stable pilot-signal in time, commuting corresponding sources in succession. For this purpose a new additional device – the modulator (or switch) – is installed before the radio-frequency amplifier. It provides alternate connection of an amplifier input to an antenna and to a special reference noise source (Figure 3.8). The reference source generates, throughout the reception band of an amplifying system, a stable noise signal that is identical in its statistical characteristics to the noise signal of the source under investigation. The most simple and reliable noise source is the radio-physical device called the matched load; this is a waveguide- or coaxial-type device, which absorbs the whole electromagnetic energy falling on it. It can easily be seen that this device is a full analogue of the black body in optics, and the intensity of its emission corresponds to the emission of an absolutely black body (Planck's function). With allowance for the Rayleigh–Jeans approximation in the microwave band, its brightness temperature is equal to the thermodynamic temperature. And, thus, the matched load placed in the thermostat represents an ideal reference noise source.

As a result of receiver input reconnections, the amplified emission is modulated with a special frequency called the modulation frequency. The correct choice of this frequency is of principal importance, of course, since it influences the efficiency of operation of the whole instrument. The reconnection frequency of a modulator is chosen to be fairly high, so that the gain cannot essentially change for one reconnection period (Figure 3.7). Apparatus studies have shown that the frequency of 1 kHz is optimum for the majority of types of amplifiers. The modulation depth depends on the difference between the received and the reference signals. This low-pass modulation of a signal conserves after quadratic detection as well. The special low-pass preamplifier selects the useful signal with a modulation frequency after detection and suppresses noise components caused by the flicker-noise and appearing after quadratic transformation (Figure 3.7). Then the signal at the modulation frequency is delivered to the special detector, which possesses phase properties (a synchronous detector) and is controlled by the reference voltage from the reference voltage generator which, in its turn, controls the operation of a modulator at the system's input. At the synchronous detector output a voltage is generated which is proportional to the difference between signals from the antenna and from a reference source, but without the presence of the flicker-noise of a high-frequency amplifier. The low-pass filter, determining the finite passband (or time constant), is used as a final integrator. In modulation radiometers the useful signal from the antenna comes to an amplifier input during the modulation half-period and, as a result, the effective sensitivity of the radiometer of such a type is twice as bad as the sensitivity of a compensatory radiometer (see below). However, the technological advantages gained in working with an instrument of this type compensates for a small loss in sensitivity.

The frequency and temporal transformations of signals in the given type of instruments are quite complicated. For this reason we shall use below both

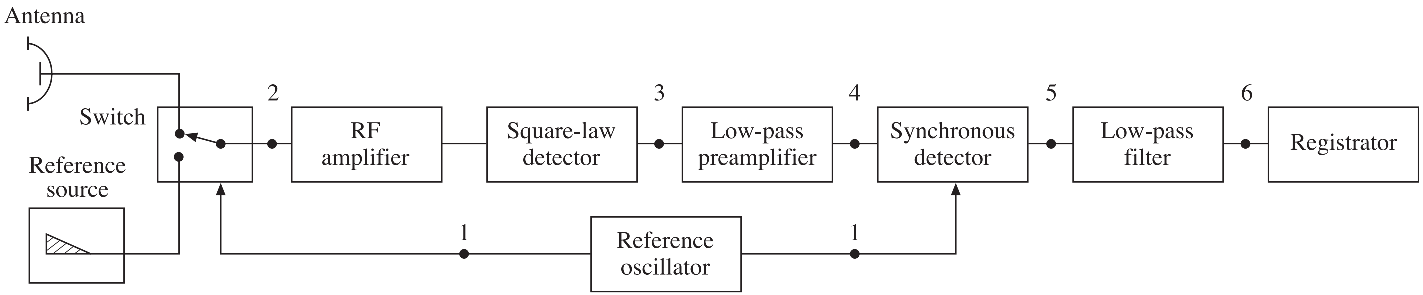

Figure 3.8. Functional scheme (block-diagram) of a switched radiometer. The places where temporal and spectral features of this scheme will be considered in detail are marked by figures.

temporal and spectral cyclograms of device operation when considering the principles of functioning of a modulation scheme.

3.6.2 Temporal and spectral cyclograms

In the block-diagram of Figure 3.8, six points are marked at which we shall consider temporal and frequency transformations of signals. Point 1 (Figure 3.9(a)) indicates the time sequence of a usually pulse signal with a frequency of 1 kHz used for controlling a modulator and a synchronous detector. The Fourier series expansion of this signal gives the following well-known expression (Gradshteyn and Ryzhik, 2000) for the amplitude–phase spectrum of a pulse signal:

$$\dot{S}(\omega) = U\frac{4}{\pi}\left(\cos\Omega t - \frac{1}{3}\cos 3\Omega t + \frac{1}{5}\cos 5\Omega t - \dots\right), \tag{3.16}$$

where U is the pulse signal amplitude, $\Omega = 2\pi f_{\mathrm{M}}$ and f_{M} is the modulation frequency. The magnitude of a power spectrum (modulo) of a signal is presented in Figure 3.9(b) (point 1). It can be seen from relation (3.16), that the amplitudes of a spectrum of symmetrical pulse sequence fall as $1/(2N+1)(N = 0, 1, 2, \dots)$, and the amplitudes of a power spectrum as $1/(2N+1)^2$. As a result of reconnection at the amplifier input, a high-frequency pulse signal with variable intensity is formed: in one half-period of the modulation frequency its intensity (rather than the amplitude) will be equal to the sum of the antenna temperature of a useful signal (under investigation) and the noise temperature of an amplifier ($T_1 = T_{\mathrm{S}} + T_{\mathrm{N}}$). In the other half-period this intensity will be equal to the sum of the reference temperature (T_0) and the noise temperature of an amplifier ($T_2 = T_0 + T_{\mathrm{N}}$). The diagram in Figure 3.9(a) (point 2) presents the qualitative form of a fluctuating amplitude of emissions whose intensities are equal to T_1 and T_2, respectively. The spectral form (we should bear in mind that Wiener spectra are being considered here) of this signal is presented in Figure 3.9(b) (point 2). The high-frequency amplifier does not change the qualitative picture (point 2) and only increases the amplitude (and, accordingly, the power) of a signal (as much as millions of times, usually). For clarity, we shall consider each modulation half-period after quadratic detection separately. Each temporal half-period will contain a constant component of a signal, which is proportional to the signal intensity in the given half-period, as well as the components of a transformed signal and flicker-noise, which are identical for each half-period (Figure 3.9(b), point 3). Considering both half-periods simultaneously, we can represent the signal obtained as a single noise signal, which will have the different mean value. Considering Figure 3.9(b) (point 3), we can easily see the important function, which should be executed by the intermediate low-pass preamplifier – (LPPA): it should suppress flicker-components, leaving untouched the components of a pulse signal whose amplitude equals the $T_1 - T_2$ (or, respectively, $T_{\mathrm{S}} - T_0$) difference. Technically this is performed as follows: the LPPA cuts off in the narrow Δf_{LPPA} band the frequency components corresponding to the pulse spectrum, and strengthens them, while suppressing all remaining frequency components, including those from flicker-noise (Figure 3.9(a) and (b), point 4). Such a

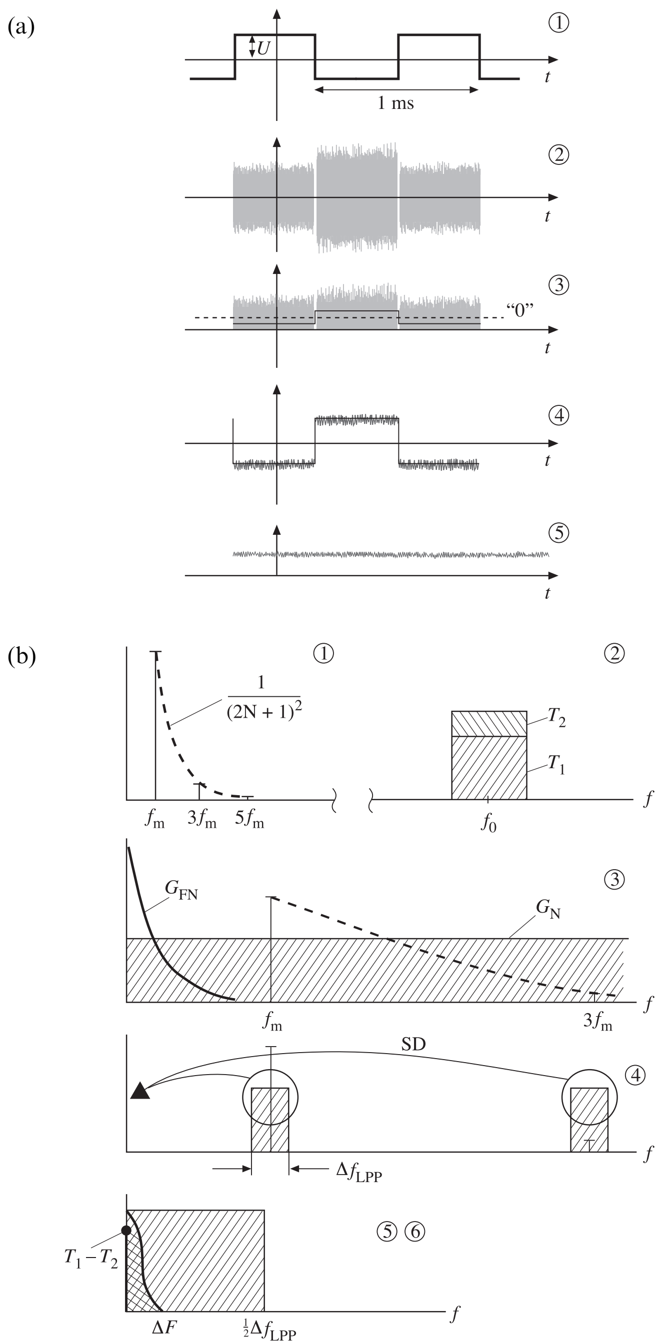

Figure 3.9. Temporal and spectral cyclograms of working for a switched radiometer. (a) Temporal cyclogram. (b) Spectral cyclogram. Figures by diagrams correspond to those places that are marked on Figure 3.8. Notation is explained in the text.

type of a filter-amplifier is called a comb filter. The next important component of a circuit is the synchronous detector. The functions of this component are very important for performing the whole signal processing procedure; therefore, we shall consider its properties in more detail below. Here we point out only the fact that the synchronous detector forms a direct current signal which is proportional to the pulse amplitude with small residual noises in its passband. This procedure is shown qualitatively in Figure 3.9(a) and (b) (point 4). Finally, the signal in the form of the $T_1 - T_2$ difference is formed by means of its passage through the low-pass filter (the integrator) (Figure 3.9(a) and (b), points 5 and 6). It can easily be seen from physical considerations that the LPPA passband has supplementary character during signal processing and should, obviously, vanish in the final result. The detailed calculations indicate that this indeed is the fact.

3.6.3 Synchronous detector

The synchronous detector is a rather critical component of the whole modulation instrument and, therefore, it is expedient to consider the basic principles of its operation. The synchronous detector is a device in which the active parameter oscillates at a frequency equal to the frequency of a delivered external signal, but with arbitrary phase, generally speaking. The simplified wiring scheme of an instrument is presented in Figure 3.10 in the form of a series connection of two active resistances R_1 and R_2, one of which is variable and controlled by the reference signal. For simplicity, we shall consider the control with a harmonic signal:

$$R_2 = R_0[1 + A\cos(\Omega t + \varphi_2)], \tag{3.17}$$

and the input voltage will also be considered as harmonic:

$$u_1 = u_0 \cos(\Omega t + \varphi_1). \tag{3.18}$$

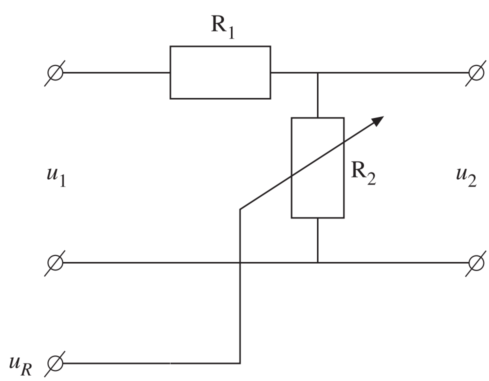

Figure 3.10. Physical presentation of the work of a synchronous detector.

Using Ohm's law and taking into account the simplifying inequality $R_1 \gg R_2$, we obtain the output voltage of the device in the form:

$$u_2 = B\frac{1}{2}\cos(\varphi_1 - \varphi_2) + \frac{B}{A}\cos(\Omega t + \varphi_1) + \frac{B}{2}\cos(2\Omega t + \varphi_1 + \varphi_2), \tag{3.19}$$

where $B = Au_0(R_0/R_1)$.

Of principal importance here is the appearance of the value of an initial signal at direct current (zero frequency) or, in other words, the performing of a strict linear detection procedure. However, the output signal will also depend on the phase difference of initial and controlling signals. In this case it can easily be seen from (3.19) that the signal maximum will be achieved at a zero phase difference, the negative signal will take place at the phase difference of π, and at the phase difference of $\pi/2$ the signal will be absent at the output of the device. Thus, the device is phase-sensitive. This property of a synchronous detector has been actively used in modulation devices both for adjustment work and for external calibration purposes (see section 3.7).

Now we shall consider the operation of this circuit at the rectangular synchronous pulsation of both an input signal itself and a controlling one. For this purpose we shall take advantage of the Fourier expansion of a rectangular pulse, presented above in (3.16), and perform a procedure similar to (3.19). In so doing the constant component of an output signal can be written as:

$$u_2 = B\frac{16}{\pi^2}\frac{1}{2}\left(1 + \frac{1}{9} + \frac{1}{25} + \dots\right). \tag{3.20}$$

The expression in brackets represents the well-known converging series, the sum of which is equal to $\pi^2/8$ (Gradshteyn and Ryzhik, 2000). Thus, the value of a numerical coefficient at constant component will be equal to unity, which is twice as great as for harmonic signals. If one of the signals is harmonic and the second is of pulse-type, then, using the same methodology, it can easily be shown that the numerical coefficient will be $2/\pi$, i.e. the intermediate value, as would be expected. For this reason, as well as according to a number of purely technological circumstances, the purely pulse regimes are used in modulation systems, both for modulation of the main input signal and for its processing at a synchronous detector.

3.6.4 Modulation reception peculiarities

The use of a reference source in modulation instruments for eliminating the parasite effect of the flicker-noise of modulated emission results, however, in some peculiarities in practical observational work. To consider these peculiarities we return to a more in-depth analysis of the signal at point 3 (Figure 3.11) on the temporal diagram. After the quadratic detector, each half-period contains a constant component proportional to T_1 and T_2, as well as the noise components including

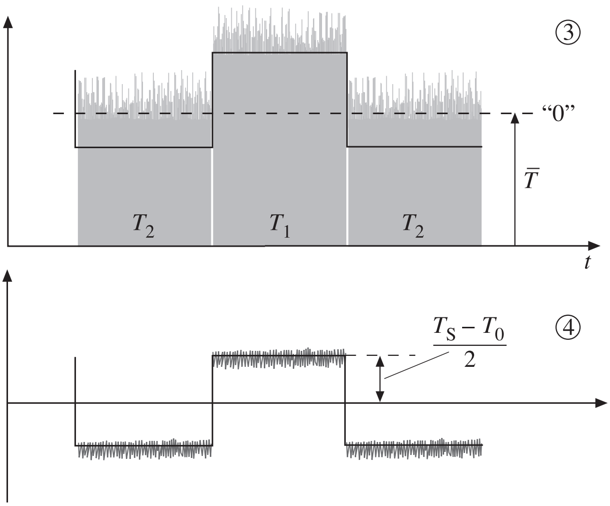

Figure 3.11. Detailed temporal cyclogram for outputs of a square-law detector (3) and a low-pass preamplifier (4).

that from flicker-noise. The constant value of a signal in each half-period can be written as follows:

$$\begin{aligned} \bar{V}_1 &= \beta\left(\overline{u_1^2}\right) = \beta k(T_N + T_S)\,\Delta f \\ \bar{V}_2 &= \beta\left(\overline{u_2^2}\right) = \beta k(T_N + T_0)\,\Delta f. \end{aligned} \tag{3.21}$$

The graphical construction in Figure 3.11 indicates, that the mean value of signal over the whole modulation period can be written as

$$\bar{T} = T_2 + \frac{T_1 - T_2}{2} = T_N + \frac{T_S + T_0}{2}. \tag{3.22}$$

As we have already noted, the most important function of the LPPA is the elimination of this constant component of a signal along with flicker-noise components which are adjacent to the constant component (see the frequency diagram in Figures 3.7 and 3.9(b)). At the LPPA output we have a symmetrical pulse signal with the amplitude proportional to $(T_1 - T_2)/2$ or $(T_S - T_0)/2$, and the fluctuation components passed through the LPPA passband. Expanding the pulse signal into the Fourier series, we shall obtain the analytical expression for a signal at the LPPA output:

$$V_{LPPA} = \beta k \frac{T_S - T_0}{2}\,\Delta f\,\frac{4}{\pi}\left(\cos 2\pi f_M t - \frac{1}{3}\cos 3\,2\pi f_M t + \frac{1}{5}\cos 5\,2\pi f_M t - \dots\right) \tag{3.23}$$

where by $A_1(t)$ are meant all fluctuation components passed through the LPPA passband. The further transformation occurs in a synchronous detector by multiplying the signal of (3.23) by a pulse signal and separating the constant component (see above). Remembering relation (3.20), we obtain the signal at the low-pass filter output V_{LPF} in the following final form:

$$V_{\mathrm{LPF}} = \frac{1}{2}\beta k(T_{\mathrm{S}} - T_0)\,\Delta f + A_1(t), \tag{3.24}$$

where by $A_1(t)$ are meant all fluctuation components passed through the low-pass filter. The analysis of the final relation (3.24) for the modulation instrument gives rise to some important features of this type of measuring devices.

First, the signal in the form of the difference between the signal under investigation and reference signal is recorded at the system's output. If these signals are equal, a zero reading is recorded at the system's output, though the signal under investigation is present at the system's input. Such a 'relative zero' in the receiving system's readings is related to the modulation scheme construction methodology. In this connection, a special type of external calibration is required in receiving systems of such a kind.

Second, the output signal of radiometric receiving systems is calibrated and normalized in the form of a scale of absolute temperature in Kelvins, reduced to the receiving system's input.

Third, because of the presence of phase-sensitive components (modulator and synchronous detector) inside a system, the output signal is also phase-sensitive with respect to the phase of the control signal for a synchronous detector. If the signals for modulator and synchronous detector are phased, then the output signal is positive with respect to the 'relative zero'. If the phase difference equals π (i.e. the signals are in the antiphase), then the signal is negative with respect to the 'relative zero'. If, however, the phase difference equals $\pi/2$, then the useful signal at the system's output is absent (the 'absolute zero'). In the following section we shall demonstrate these features for the example of the calibration of a radiometer.

3.6.5 Threshold sensitivity

As we have noted above, the threshold sensitivity of a receiving system is determined by those noise components that have passed right through a system after quadratic transformation. The spectrum of noise components after quadratic transformation is fairly complicated: it includes both the components from the noise of an instrument and the flicker-noise components (Figure 3.9(b), point 3). The LPPA passes only the components from the amplifier noise linked to harmonic components of a modulation pulse. Since the system under consideration is linear, for calculating the intensity of output (with respect to LPPA) fluctuations we shall consider the signal correlation function at the LPPA output taking into account the fact that the amplitude-spectral characteristics of the LPPA (in power) represents a comb filter (Figure 3.9(b), point 4) in the form of:

$$B_{\mathrm{LPPA}}(\tau) = \sum_{N=0}^{\infty} \int_{(2N+1)\omega_{\mathrm{M}}-(\Delta\omega_{\mathrm{M}}/2)}^{(2N+1)\omega_{\mathrm{M}}+(\Delta\omega_{\mathrm{M}}/2)} G_{\mathrm{N}}(\omega)\cos(2N+1)\omega\tau\,\mathrm{d}\omega, \tag{3.25}$$

where $G_N(\omega)$ is the low-frequency component of quadratic transformation (see section 2.8 and relation (2.92)), $\omega_M = 2\pi f_M$, $\Delta\omega_M = 2\pi\Delta f_M$. As we have already noted, the relationship between the modulation frequency and high-frequency band of an amplifier is such that $f_M \ll \Delta f$. Then we can let $G_N(\omega) = G_N(0)$ (see Figure 3.9(b), points 3 and 4). Besides, in order to almost completely reproduce the pulse signal, it is sufficient to pass, for example, N of its harmonics ($N = 5$–7). Thus, for N and f_M values that are not too high the condition of $Nf_M \ll \Delta f$ conserves for the highest harmonic, and in relation (3.25) the $G_N(\omega) = G_N(0)$ condition can be conserved. After performing integration, we can get

$$B_{LPPA}(\tau) = 2\beta^2 \frac{\sigma^4}{\Delta\omega} \frac{\sin \dfrac{\Delta\omega_M \tau}{2}}{\dfrac{\Delta\omega_M \tau}{2}} \sum_{N=0}^{\infty} \cos(2N+1)\omega_M \tau, \tag{3.26}$$

where

$$\sigma^2_{LPPA} = \overline{V^2_{LPPA}} = 2\beta^2 \frac{\sigma^4}{\Delta\omega} \Delta\omega_M$$

is the variance (intensity) of the fluctuation signal $V_{LPPA}(t)$ at the LPPA output. The signal formed at the LPPA output comes to the input of a synchronous detector and is multiplied by the reference signal having symmetrically rectangular form:

$$V_{SD}(t) = V_{LPPA}(t) \frac{4}{\pi} \left(\cos\omega_M t - \frac{1}{3}\cos 3\omega_M t + \frac{1}{5}\cos 5\omega_M t - \dots \right). \tag{3.27}$$

Now we determine the correlation function of a signal at the synchronous detector output, making use of ergodic properties of the process under consideration (see section 2.2):

$$\begin{aligned} B_{DS}(\tau) &= \overline{V_{SD}(t)\, V_{SD}(t+\tau)} = \overline{V_{LPPA}(t)\, V_{LPPA}(t+\tau)} \\ &\quad \times \overline{\frac{16}{\pi^2} \left(\cos\omega_M t - \frac{1}{3}\cos 3\omega_M t + \dots \right) \left(\cos\omega_M (t+\tau) - \frac{1}{3}\cos 3\omega_M (t+\tau) + \dots \right)}. \end{aligned} \tag{3.28}$$

The mean value of two latter series multiplied by each other can be written as

$$\sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \cos(2N+1)\omega_M \tau. \tag{3.29}$$

The complete expression for the correlation function is fairly complicated. For our purposes it is sufficient to separate the largest-scale component, which is determined as a constant component in the time-averaging of a product of two series, namely,

$$\sum_{N=0}^{\infty} \cos(2N+1)\omega_M \tau \sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \cos(2N+1)\omega_M \tau. \tag{3.30}$$

Making the necessary trigonometric transformations, we shall obtain the expression for a constant component of the product of these series:

$$\frac{1}{4}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)=\frac{\pi^2}{32}. \tag{3.31}$$

Thus, the correlation function of a signal after synchronous detector will contain both a large-scale and a small-scale component, which will be designated by $B(\tau, \omega_{\mathrm{M}})$:

$$B_{\mathrm{SD}}(\tau)=\frac{1}{2}\overline{V_{\mathrm{LPPA}}^2}\,\frac{\sin\dfrac{\Delta\omega_{\mathrm{M}}\tau}{2}}{\dfrac{\Delta\omega_{\mathrm{M}}\tau}{2}}+B(\tau,\omega_{\mathrm{M}}). \tag{3.32}$$

Of interest to us is the first item, which determines the part of the spectrum of a signal at the synchronous detector output directly adjacent to a zero frequency. For this purpose, remembering the relation for the spectral density for positive frequencies $G^+(f)=2\pi G(2\pi f)$ and the relationship between the correlation function and spectral density (2.26), we obtain:

$$G^+(f)=4\frac{1}{2}\overline{V_{\mathrm{LPPA}}^2}\int_0^\infty\frac{\sin\dfrac{\Delta\omega_{\mathrm{M}}\tau}{2}}{\dfrac{\Delta\omega_{\mathrm{M}}\tau}{2}}\cos 2\pi f\tau\,\mathrm{d}\tau. \tag{3.33}$$

The integral can be taken (Gradshteyn and Ryzhik, 2000) under the condition of $0<f<\Delta f_{\mathrm{M}}/2$, which is usually satisfied under particular conditions. Thus, taking into account (3.26), we have

$$G^+(f)=2\beta^2\frac{\sigma^4}{\Delta f}, \tag{3.34}$$

here Δf is the passband of the basic high-frequency amplifier. Then the variance of a fluctuation signal at low-pass filter's output will be equal to

$$\sigma_{\mathrm{LPF}}^2=\int_0^{\Delta F}G^+(f)\,\mathrm{d}f=2\beta^2\sigma^4\frac{\Delta F}{\Delta f}. \tag{3.35}$$

From this relation it is possible to find, with regard to (3.26), the reference deviation (RMS) of the fluctuation signal at an output of the whole receiving system

$$\sigma_{\mathrm{LPF}}=\sqrt{2}\beta k T_{\mathrm{N}}\,\Delta f\sqrt{\frac{\Delta F}{\Delta f}}. \tag{3.36}$$

And, at last, using the conditions of determination of the threshold sensitivity of a receiving system (ΔT) and comparing (3.35) and (3.24), where we let $T_{\mathrm{S}}-T_0=\Delta T$, we obtain the value of the threshold sensitivity for the modulation receiver:

$$\Delta T=2\sqrt{2}T_{\mathrm{N}}\sqrt{\frac{\Delta F}{\Delta f}}. \tag{3.37}$$

As should be expected, the LPPA band value dropped out from the final result and the sensitivity of the modulation radiometer was found to be twice as bad as the radiometer of a compensatory scheme. This is explained physically by the fact that due to modulation at the system's input the signal under investigation is present at the instrument's input during half of the observation time only. This implies that the power of a useful signal is two times lower than in the case of a compensatory radiometer, under identical noise properties of a receiving system. However, as we have noted, in this case the modulation scheme possesses serious advantages as compared to the compensatory (ideal) scheme.

For other forms of modulation and demodulation (synchronous detection) the threshold sensitivity will be slightly worse, in virtue of the decrease in transformation coefficients we considered earlier. So, for sinusoidal modulation and demodulation the threshold sensitivity will worsen (with respect to the ideal scheme) – by as much as 2.82 times. By these reasons, rectangular modulation and demodulation is used in the majority of systems for the reception of weak noise radiation.

The detailed analysis of the operation of various schemes of fluctuation radiation reception, apart from those considered above – compensatory and modulation (Esepkina *et al.*, 1973) – has shown that, for identical noise and band parameters of a receiving system, the schemes of radiometric systems possess a threshold sensitivity value in the form of relation:

$$\Delta T = \alpha T_{\mathrm{N}} \sqrt{\frac{\Delta F}{\Delta f}} = \frac{\alpha}{2} \frac{T_{\mathrm{N}}}{\sqrt{\Delta f\, \tau}}, \tag{3.38}$$

where α is the coefficient determining the efficiency of operation of a particular scheme. The compensatory scheme (with $\alpha = \sqrt{2}$) has the best sensitivity. All the remaining schemes give the α value in the range from 2 to 4. The reader can consult the detailed analysis of various schemes in the specialized radio-astronomical and radiophysical literature (see, for example, Esepkina *et al.*, 1973).

3.6.6 Null-balancing type

The detailed examination of expressions (3.23) and (3.8) indicates that the essential part of an output signal of the modulation radiometer can be presented in the following form:

$$V_{\mathrm{LPF}} = \tfrac{1}{2} k_1 G_{\mathrm{A}} k (T_{\mathrm{S}} - T_0)\, \Delta f. \tag{3.39}$$

It can be seen from this expression that flicker-noise can still influence the output signal of a modulation instrument, though in a highly suppressed form. Using the methodology of calculating the threshold sensitivity of a radiometric device with an allowance for flicker-noise, considered above, and taking into account (3.39), we obtain the expression for the threshold sensitivity of a modulation radiometer with allowance for flicker-noise:

$$\Delta T = \sqrt{T_N^2 \frac{\Delta F}{\Delta f} + (T_S - T_0)^2 \left(\frac{\sigma_{FN}}{G_A}\right)^2}. \tag{3.40}$$

It can be seen from this expression that the flicker-noise effect will be determined by the relation: $(T_S - T_0)/T_N$. If in real observations this parameter is essentially smaller than unity, then we can hope for virtually full compensation and elimination of the flicker-noise effect on measurement results. However, in the regime of onboard measurements of the thermal radiation of the Earth's surface, where the natural background radiation can vary within considerable limits – from 150 K (water surfaces) up to 300 K (forest massifs) – the value of the considered parameter can be significant (from 0.1 to 0.5). And in this case a virtually uncontrollable flicker-noise contribution can already be very noticeable. To exclude this offensive effect, about thirty years ago (1972–1974) the null-balancing schemes of observations in modulation radiometers were proposed both for ground-based and for onboard observations (see Chapter 14). The physical essence of this method consists in the additional introduction of a noise signal into the signal channel. This signal does not differ in its statistical characteristics from signal characteristics and has such intensity that the sum of signals is precisely equal to the value of the reference temperature throughout the measurement cycle. Such a type of measurements is called the null-balancing scheme of measurements. It can easily be seen from (3.40) that such a technique provides a full pay-off in sensitivity for the modulation scheme of measurements. This type of measurements is illustrated schematically in Figure 3.12(a). Almost instantaneous balance between the reference temperature and the antenna temperature has been achieved with the help of an additional noise generator, whose intensity is controlled by the special feedback circuit, for producing the so-called 'noisy regime' in a signal circuit. The use of such a regime makes it possible to fairly reliably distinguish and record radiobrightness contrasts up to 0.1 K and lower against the brightness background of the water surface (150 K) (see Chapter 12). Other modifications of a null-balancing regime are also possible. As an example, Figure 3.12(b) shows the input part of a modulation radiometer with controlled noise reference temperature. The horn antenna of the reference channel is pointed to the zenith, thus ensuring the reception of stable radio-emission of the atmosphere at the level of tens of absolute degrees with subsequent addition of radiation (if necessary) from the controlled noise source to provide a null-balancing regime. Such an instrument scheme is used, as a rule, in studying low-noise physical objects, such as the Earth's atmosphere in the centimetre wavelength band, the relic background of the universe, water surfaces with strong water mineralization, and artificial metal surfaces and similar physical objects, which are very 'cold' in the radiothermal respect.

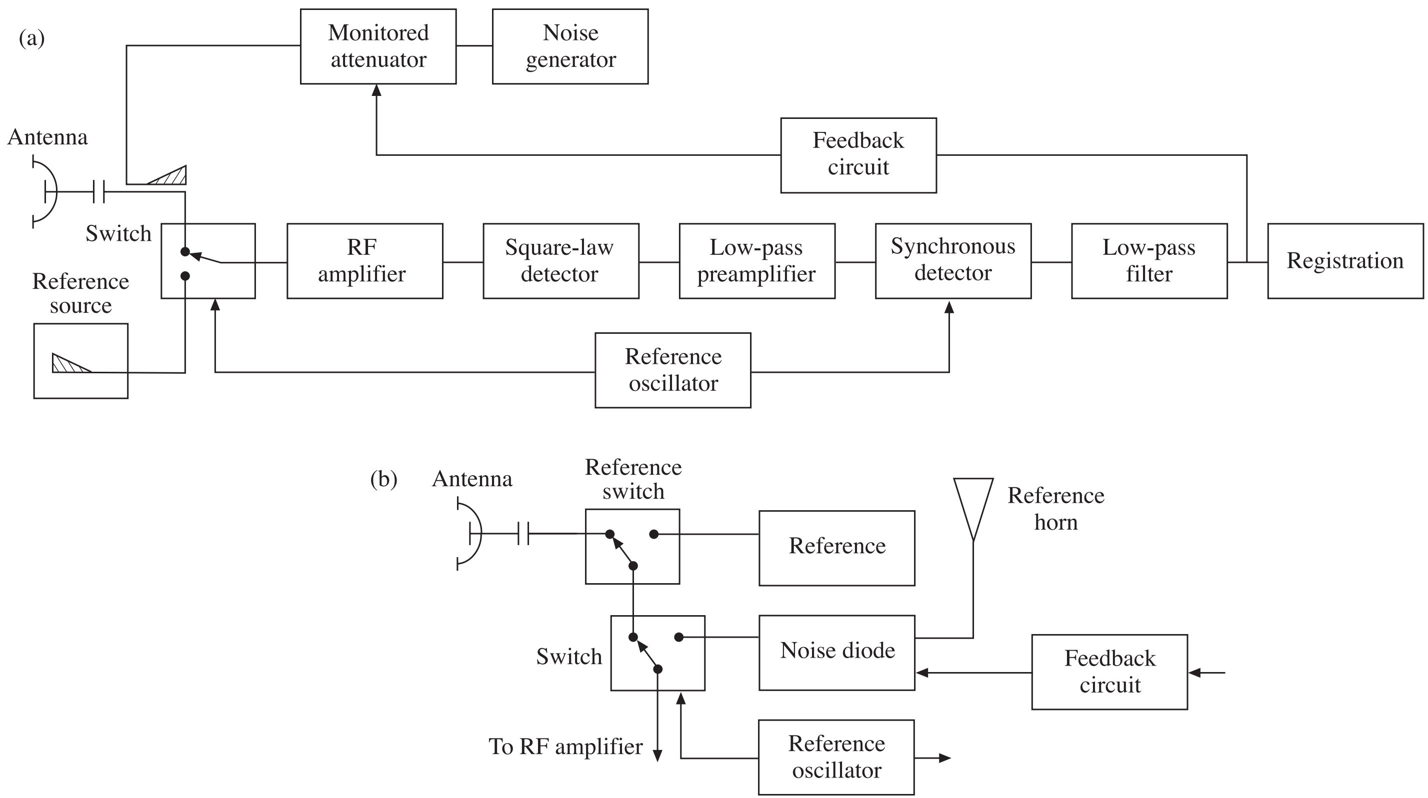

Figure 3.12. Block-diagram of a null-balancing radiometer. (a) The null-balancing scheme with noise balancing in the input circuit. (b) The scheme with noise balancing in the reference circuit.

3.7 EXPERIMENTAL METHODS OF THRESHOLD SENSITIVITY MEASUREMENT

The previous analysis suggests that the basic parameters of a radiometer, which determine its threshold sensitivity, are noise and frequency responses of the main radio-frequency amplifier and the time constant of an output integrator (the low-pass filter). Knowledge of these parameters makes it possible to calculate the expected threshold sensitivity from relation (3.38). In the process of development and adjustment of radiometric complexes a set of measurement procedures has to be performed. They include: the measurement of losses in the input channel and in its separate components; the contribution of separate cascades to the general noise temperature; and the frequency responses of separate cascades and the compatibility of their frequency properties. There is a rich special radio-engineering literature devoted to the theory and practice of such fine measurements. Here we shall propose some practical recommendations for the measurement and estimation of the basic parameters of a radiometric complex, the threshold sensitivity first of all. In observational practice technological circumstances frequently arise in which it becomes necessary to rapidly and reliably check the threshold sensitivity of a radiometric system. As we have noted above, the generally accepted criterion for determining the threshold sensitivity of a radiometric system is the equality of a minimum noise signal at the system's input to the reference deviation (RMS) of output fluctuations of the system (see relations (3.10) and (3.38)). Thus, for experimental determination of the threshold sensitivity it is necessary to record, with the help of a self-recorder or digital recorder, the output signal of a radiometric system with no signal at an input. The recording should be performed at a large enough scale in both axes – of signal and time – so that the fluctuation character (the Gaussian noise) of an output signal may be clearly seen, and then to deliver a calibrating noise signal of known magnitude to an input of the whole receiving system. As a result, we obtain on the self-recorder's tape the record, in the form of a peculiar 'noisy zigzag path', of the normal random process (see Chapter 2) with a calibrating stepwise signal. Thereby, the whole receiving system and its recording section will be calibrated and reduced to the receiving system's input. As an indicative example, Figure 3.13 presents the registration recording of an output fluctuation signal with calibration signals of an onboard high-sensitivity modulation radiometer of 8-millimetre wavelength band (Militskii *et al.*, 1975). All calibration procedures demonstrated below were performed under flight conditions aboard the Russian IL-18 airplane-laboratory in 1975 (see Chapter 14).

In accordance with statistical procedures (Bendat and Pirsol, 1966; Esepkina *et al.*, 1973; Cressie, 1993), the reference deviation can be determined from the experimental data as follows. The readings of output signal a_i are counted from some conventional zero in regular time intervals Δt. It is important that the intervals be (4–5 times) greater than the time constant of an output integrator (τ), since otherwise the neighbouring values of readings will not be independent, and this circumstance can essentially distort the sought result. It is also important to be sure that the 'pure' Gaussian random signal of a receiving system is present at the system's output, because the application of analogue-to-digitial converters, which is widespread now, seriously distorts the original statistics of a signal (Bendat and Pirsol, 1966). An experienced experimenter can usually determine the 'purity' of a signal as a Gaussian one from the form of a noise signal recording. Further on, the mean value, the squares of deviation from the mean value and, finally, the reference (unbiased) deviation are found from n readings (Bendat and Pirsol, 1966) as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (\bar{a} - a_i)^2}{n - 1}}, \tag{3.41}$$

where $\bar{a}$ is the mean value. Further, using the known calibration signal, we find the price of an output scale: $k = T_k/a_k$, and then we find the value of a reference deviation (the threshold sensitivity), in absolute degrees, reduced to the receiving system's input, $\Delta T = k\sigma$. The procedure is fairly lengthy, in general, since it requires the digitization of a great number of readings. If the system contains a digital processor in its design, then the aforementioned procedure can be performed automatically and regularly, in accordance with the programme of observations.

As we have noted above, the reference deviation magnitude, measured from a limited realization (the n value) can noticeably differ from the limiting (true) reference deviation value. Experimenters often (and rather successfully) use the other approach associated with the 'three sigma rule' for the normal distribution (see section 2.2). The total ('pick-to-pick') width of noise signal recording for a considerable time interval is determined, in essence, by the probability of the random signal escaping the prescribed interval. Thus, comparing the total pick-to-pick with the price of a scale of an output signal and calculating one-sixth of this magnitude, we obtain a value for the threshold sensitivity of a radiometric system that is sufficient (for practical purposes) and reliable (to a satisfactory accuracy of 15–20%). Figure 3.13(a) demonstrates this procedure for two time constants, 0.25 and 1 sec. The threshold sensitivity has changed (improved), as would be expected, by as much as twice. The next diagram, Figure 3.13(b), presents the registration of an output signal for the same instrument, but with a time constant of 4 sec and two velocities of motion of the self-recording instrument's tape: the left-hand part at a velocity of 1800 mm/hour, and the right-hand one at 240 mm/hour. This procedure is specially performed to demonstrate the characteristic form of the transition exponential process of an output integrator, the full time of which is, as known, of the order of 5τ, or, for the given version of a scheme, 20 sec (the left-hand half of Figure 3.13(b)). The characteristic correlation of signal fluctuations (the 'smooth' signal) is also revealed on the noise track (pick-to-pick) over time scales shorter than 5τ. As the tape motion velocity decreases (or the temporal signal accumulation increases), the character of the signal recording becomes close to the random Gaussian signal (the right-hand part of Figure 3.13(b)). The threshold sensitivity of an instrument lowers twice more. Figure 3.13(c) demonstrates, for the same instrument, the procedures of finding the 'relative zero' of the instrument's output scale and the change of phase (by π) of a controlling signal by a synchronous detector. For this purpose the calibrated source of a noise signal – the matched

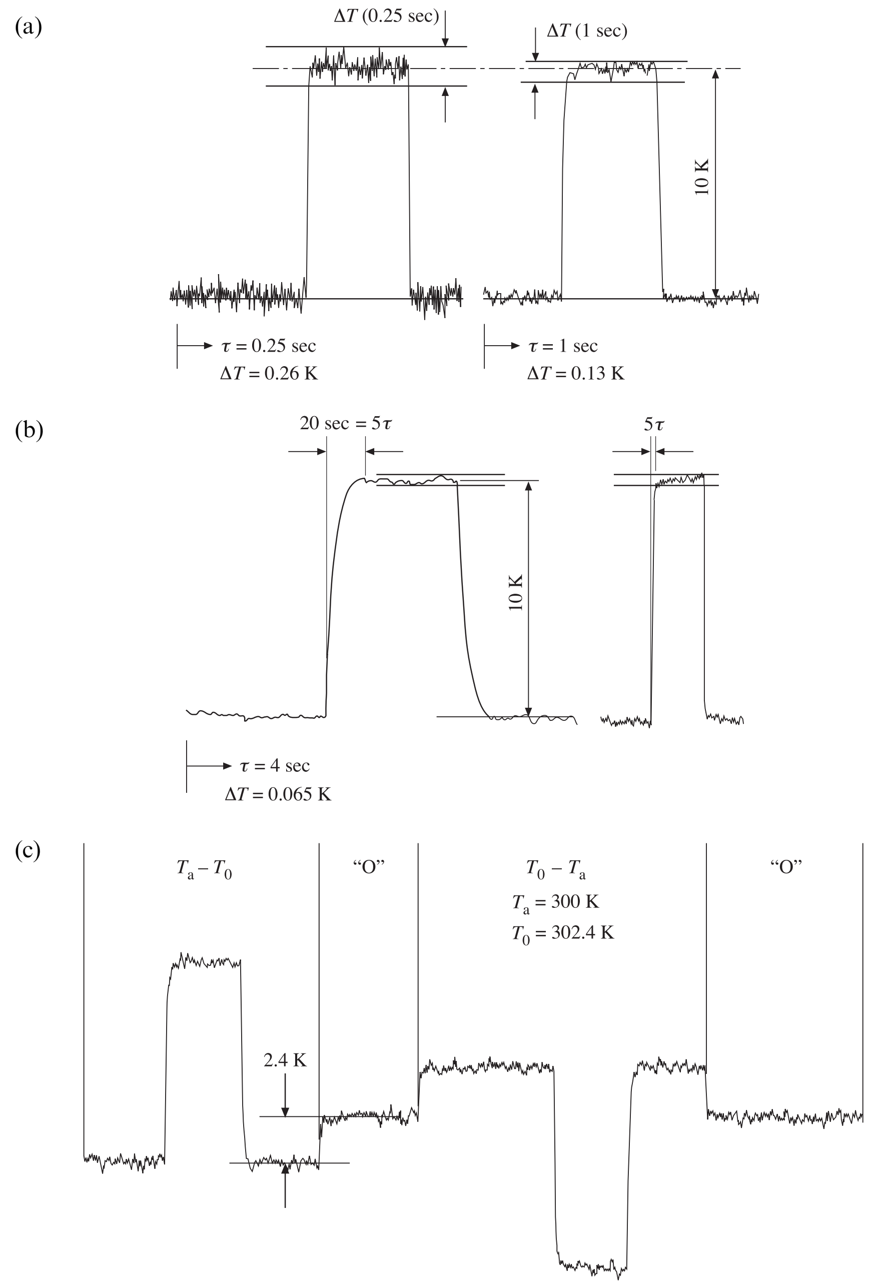

Figure 3.13. Experimental registrogam of an output fluctuation Gaussian signal for calculating the radiometer threshold sensitivity using a 'step input' signal equal to 10 K: (a) for two time constants – 0.25 s and 1 s; (b) for 4 s and for two recording velocities – 1800 mm/h (left) and 240 mm/h (right); (c) the records of a step input signal for two various phases of modulation signal to a synchronous detector. Notation is explained in the text.

load with a fixed thermodynamic temperature of 300 K – was connected to the basic signal channel, and the recording of a signal was performed with a calibrating impulse of 10 K (this part of registration is designated as $T_a - T_0$). To check the accuracy of the reference temperature value and the correctness of setting the 'relative' zero, the following procedure was performed. While the synchronous detector was operating, the modulator was set at the fixed position (to the signal channel), and the recording of the signal was accomplished. In this case the signal represented the difference of two identical quantities (T_0) and was, accordingly (conventionally speaking, of course), an 'absolute' zero for the output signal scale. This part of the registration is designated as 'O' in Figure 3.13(c). The analysis of the registration clearly shows that the merging of two 'zero' records took place at 2.4 K (from the calibration step data). Since the input signal (the matched load) was carefully calibrated in advance (300 ± 0.1 K), the mentioned effect is due to the fact that the value of a signal from the reference source was 302.4 K (rather than 300 K, as was supposed earlier). The next part of the recording, designated as $T_0 - T_a$, was performed by changing the phase (by π) of the control signal on the synchronous detector and at the previous phase of the controlling voltage on the modulator. It can easily be seen that, with respect to the 'absolute' zero, this recording is mirror-symmetrical to the $T_a - T_0$ recording, which is just as would be expected, strictly speaking. Thus, the example considered above has demonstrated the experimental (under onboard conditions) capabilities of determining: the threshold sensitivity of a radiometric instrument; its time constant; the calibration and setting of its output signal scales; and correcting the reference signal value.

3.8 MEASUREMENT OF THE FREQUENCY RESPONSES OF RADIOMETRIC SYSTEMS BY FOURIER SPECTROSCOPY METHODS

Measurement of the frequency responses of linear amplifying systems are carried out by well-known radiotechnological measurement procedures. However, unlike determinate signals, fluctuation electromagnetic radiation, while possessing a very broad thermal radiation spectrum, occupies the whole passband of a receiving system, including the far wings of the passband. The signal, formed by the whole of this band, is just subject to quadratic transformation. As a result, the effective signal at the quadratic converter output will be proportional to the integral of the square of the power frequency response of the amplification cascade. In order to show this for the frequency response of arbitrary form $G_A(\omega)$, we shall write the expression for the spectral density of a noise signal at an amplifier's output $G_N(\omega)$, with regard to the matching condition at its input (see section 3.3 and equations (3.8) and (3.9)), as follows:

$$G_N(\omega) = kT_N G_A(\omega). \tag{3.42}$$

In accordance with (2.23) and (2.27), we shall obtain the expressions for the correlation function and the value of the signal variance at the amplifier's output in the form of:

$$B_{\mathrm{N}}(\tau)=\tfrac{1}{2}kT_{\mathrm{N}}\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega)\exp(\,\mathrm{j}\omega\tau)\,\mathrm{d}\omega \tag{3.43}$$

$$\sigma_{\mathrm{N}}^{2}=B_{\mathrm{N}}(0)=\tfrac{1}{2}kT_{\mathrm{N}}\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega)\,\mathrm{d}\omega. \tag{3.44}$$

Based on these relations, we can obtain the expression for the coefficient of noise signal correlation at an output, presented using frequency response of an amplifier, as follows:

$$R_{\mathrm{N}}(\tau)=\frac{B_{\mathrm{N}}(\tau)}{\sigma_{\mathrm{N}}^{2}}=\frac{\displaystyle\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega)\ \exp\,(\,\mathrm{j}\omega\tau)\,\mathrm{d}\omega}{\displaystyle\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega)\,\mathrm{d}\omega}. \tag{3.45}$$

Certainly, all the relations obtained can also be written down using the positive frequencies and corresponding frequency response $G_{\mathrm{A}}^{+}(f)$ only. So, the expression for the correlation coefficient will be:

$$R_{\mathrm{N}}(\tau)=\frac{\displaystyle\int_{0}^{\infty}G_{\mathrm{A}}^{+}(f)\cos 2\pi f\tau\,\mathrm{d}f}{\displaystyle\int_{-0}^{\infty}G_{\mathrm{A}}^{+}(f)\,\mathrm{d}f}. \tag{3.46}$$

Now we shall write the correlation function of a signal after quadratic transformation (2.88) in the form more convenient for further analysis:

$$B_{\mathrm{SLD}}(\tau)=\sigma_{\mathrm{N}}^{4}+2B_{\mathrm{N}}^{2}(\tau). \tag{3.47}$$

(For simplicity of expression we let the value $\beta=1$.)

As we have noted, the spectral composition of a signal after quadratic transformation consists of two components: the information component disposed at direct current (a zero frequency), and the component responsible for noise components of a signal. The spectral density of the latter components $G_{\mathrm{SLD}}(\omega)$ can be expressed via the correlation function of a signal at the amplifier's output as follows:

$$G_{\mathrm{SLD}}(\omega)=\frac{2}{\pi}\int_{-\infty}^{\infty}B_{N}^{2}(\tau)\exp\,(-\mathrm{j}\omega\tau)\,\mathrm{d}\tau. \tag{3.48}$$

Making use of relations (2.23) and (2.24), we write down the integral in equation (3.49) as follows:

$$\begin{aligned}\int_{-\infty}^{\infty}B_{\mathrm{N}}^{2}(\tau)\exp\,(-\mathrm{j}\omega\tau)\,\mathrm{d}\tau&=\frac{kT_{\mathrm{M}}}{2}\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega')\,\mathrm{d}\omega'\int_{-\infty}^{\infty}B_{\mathrm{N}}(\tau)\exp\,(-\mathrm{j}\tau(\omega-\omega'))\,\mathrm{d}\tau\\&=\frac{(kT_{\mathrm{N}})^{2}\,\pi}{2}\int_{-\infty}^{\infty}G_{\mathrm{A}}(\omega')G_{\mathrm{A}}(\omega-\omega')\,\mathrm{d}\omega'.\end{aligned} \tag{3.49}$$

Substituting this relation into (3.48), we obtain the principal expression relating the spectral density of noise components of a signal at the quadratic detector's output with the integral of convolution of the amplitude–frequency characteristic of the amplifier of arbitrary form:

$$G_{\mathrm{SLD}}(\omega) = (kT_{\mathrm{N}})^2 \int_{-\infty}^{\infty} G_{\mathrm{A}}(\omega')G_{\mathrm{A}}(\omega - \omega')\, \mathrm{d}\omega'. \tag{3.50}$$

In this case the maximum value of spectral density, determining noise components of the detected signal, will be equal (for $\omega = 0$) to the following value:

$$G_{\mathrm{SLD}}(0) = (kT_{\mathrm{N}})^2 \int_{-\infty}^{\infty} G_{\mathrm{A}}^2(\omega)\, \mathrm{d}\omega. \tag{3.51}$$

Note once again that the maximum value of the spectral density of a detected signal is proportional to the integral of the square of the amplitude–frequency characteristic of a power amplifier. Earlier we obtained the expression from the spectral density for the rectangular band characteristic (2.92) in another way, namely, using the Fourier transformation from a calculated correlation function. It should be noted that, since the rectangular band characteristic cannot be realized physically, the aforementioned results are usually considered as an important model-limiting case. However, in the case of real amplifying devices, expressions (3.50) and (3.51) can be rather complicated. For this reason it was decided, that it would be worthwhile to introduce (Bunimovich, 1951; Esepkina *et al.*, 1973) the notion of the equivalent low-pass band of detected noises (or the radiometric band of an instrument) (Δf_{RAD}) as the ratio of the total power of noise components (at the detector's output) to the maximum value of spectral density. Passing to positive frequencies and to the real amplitude–frequency characteristics of an instrument $G^{+}(f)$, we shall have:

$$\Delta f_{\mathrm{RAD}} = \frac{2\sigma^4}{G^{+}(0)} = \frac{\left[\int_0^{\infty} G^{+}(f)\, \mathrm{d}f\right]^2}{\int_0^{\infty} [G^{+}(f)]^2\, \mathrm{d}f}. \tag{3.52}$$

It can easily be seen from (3.52) that, for the rectangular band, the radiometric band value will be equal to the full absolute value of a band. In the case of other forms of amplitude–frequency characteristics, however, the situation can radically differ from such an idealized (and unfeasible) case (Bulatov *et al.*, 1980).

Remembering the threshold sensitivity determination procedure (see section 3.5), we equate the value of the square of a constant component at a detector's output, determined by the input signal equal to the threshold sensitivity, to the value of variance of a noise signal at the output of a low-pass filter with the effective band ΔF:

$$\sigma^4(\Delta T) = G^{+}(0)\, \Delta F. \tag{3.53}$$

After some transformations we shall have the value of the threshold sensitivity expressed via the radiometric band of a high-frequency amplifier and the effective band of a low-pass filter:

$$\Delta T = \sqrt{2}\, T_{\mathrm{N}} \sqrt{\frac{\Delta F}{\Delta f_{\mathrm{RAD}}}}. \tag{3.54}$$

Since real amplifying devices sometimes have fairly complicated amplitude–frequency characteristics, the necessity arises for a comprehensive and quite simple (in respect of instrumentation) investigation technique. The material presented below is based on the results of experimental works performed at the Space Research Institute under the supervision of the author of this book in 1976–1978 (Bulatov *et al.*, 1980). The physical essence of these techniques is as follows. First, the artificial interference of broadband noise signals is generated at the input of a tested radiometric system; then, the interference signal (the Fourier spectrogram, in essence) is recorded at the output of a radiometric device; and, finally, the total appearance and form of the amplitude–frequency characteristic of the instrument are restored. The principal diagram of the experiment and radiometric instrument is presented in Figure 3.14. The broadband noise signal from the noise generator (conventionally, delta-correlated) is divided in half and delivered to the radiometer's input over circuits of various electrical length. At the radiometer's input, two noise signals, shifted in time relative to each other, are intermixed and enter the input of the amplifier itself with a complicated form of a restricted amplitude–frequency characteristic, which will just 'impose' correlation links on the input signal. It can easily be seen from comparison of Figures 3.2(d) and 3.14, that the experimental scheme used is, in essence, an analogue of Michelson's interferometer scheme and can be used for Fourier spectrometry purposes.

Slightly simplifying the situation, we shall consider the interference of noise signals in the so-called 'quasi-monochromatic' approximation. By changing the difference of circuit arms $l = l_1 - l_2$, the interference of harmonic signals E and $E \exp(-jkl)$ takes place at the radiometer's input, where $k = 2\pi/\lambda$. The sum of fields at an input is equal to $E_{\mathrm{in}} = E(1 + \exp(-jkl))$, and the power in a narrow band of frequencies, df (in other words, the spectral density), is equal to

$$\mathrm{d}P \approx E_{\mathrm{in}} E_{\mathrm{in}}^{*}\, \mathrm{d}f = 2E^2(1 + \cos kl)\, \mathrm{d}f \tag{3.55}$$

and, in accordance with (3.5), it is proportional to the input antenna temperature considered in the monochromatic approximation. In this case, the response of a radiometric device (T_Σ), considered throughout the band of an instrument and reduced to the radiometer's input, can be presented as:

$$T_\Sigma = \frac{\displaystyle\int_0^\infty T_{\mathrm{N}}(f) G^{+}(f)\, \mathrm{d}f}{\displaystyle\int_0^\infty G^{+}(f)\, \mathrm{d}f}, \tag{3.56}$$

where

$$T_{\mathrm{N}}(f) = \frac{a_0}{2} + a_1 \cos 2\pi f \tau; \tau = \frac{l}{c}.$$

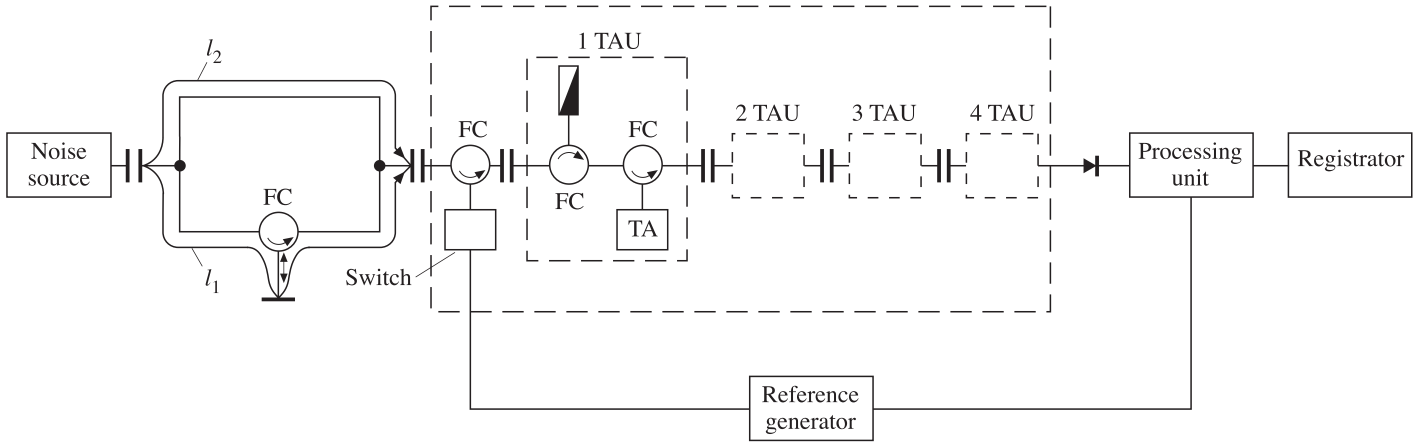

Figure 3.14. Block-diagram of the measurement scheme and the switched radiometer. TAU is a tunnelling amplifier unit; FC is a ferrite circulator (the single-acting device); TA is a tunnelling amplifier (Bulatov *et al*., 1980).

Substituting this expression into (3.56), we obtain the expression for the system's response in the form of

$$T_{\Sigma}(\tau) = \frac{a_0}{2} + a_1 \frac{Q(\tau)}{\int_0^{\infty} G^{+}(f)\, \mathrm{d}f}, \tag{3.57}$$

where $Q(\tau)$ can be presented as

$$Q(\tau) = \int_0^{\infty} G^{+}(f) \cos(2\pi f \tau)\, \mathrm{d}f. \tag{3.58}$$

It can easily be concluded from this result that for continuous change of the delay between the signals we obtain the experimental registration of a correlation function of an amplitude–frequency characteristic (AFC) of the system's amplifier. Taking into account that the passband of a receiving device is fairly narrow, function $Q(\tau)$ can be presented as a slowly varying rounding (envelope shape), which determines the AFC form, and harmonic filling, which determines the central AFC frequency (see section 2.5):

$$Q(\tau) = q(\tau) \cos 2\pi f_0 \tau. \tag{3.59}$$

Such a separation of the experimental function $Q(\tau)$ into high-frequency filling and rounding (envelope shape) is a very convenient approach, which can be fairly easily performed under experimental conditions (see Figure 3.15(a)). It essentially makes it possible to facilitate further calculations. For symmetrical passbands the form of an amplitude–frequency characteristic in power can be restored (for positive frequencies) as follows:

$$G^{+}(F) = 4\int_0^{\infty} q(\tau) \cos(2\pi F \tau)\, \mathrm{d}\tau. \tag{3.60}$$

where $F = f - f_0 > 0$.

Now we consider these procedures for the aforementioned example of the interference pattern data for a direct amplification radiometer (Figure 3.15(a)). To restore the form of the AFC and determine its parameters, the experimental values of a rounding were smoothed by the least squares method and then approximated by the function of form

$$q(\tau) = K(1 + M\tau) \exp(-m\tau), \tag{3.61}$$

where M and m in the case under investigation were equal to $(7.5 \pm 0.3) \times 10^8\ \mathrm{sec}^{-1}$ and $(12.3 \pm 0.4) \times 10^8\ \mathrm{sec}^{-1}$, respectively. Using expression (3.60), we obtain, after some integral transformations, the analytical expression for the form of the AFC of the amplifying system under investigation:

$$G^{+}(f - f_0) = 4K\left\{\frac{m}{m^2 + 4\pi^2(f - f_0)^2} + M\frac{m^2 - 4\pi^2(f - f_0)^2}{[m^2 + 4\pi^2(f - f_0)^2]^2}\right\}. \tag{3.62}$$

The form of the AFC obtained (positive frequencies) is presented in Figure 3.15(b), from which we can find the total passband from the level of 3 dB – $\Delta f_{1/2} = 272 \pm 8$ MHz, which to an accuracy of measurement error coincides with the passband measured independently from a sweep-generator (265 ± 10 MHz). The form of the amplifying system's AFC differs in principle, of course, from the

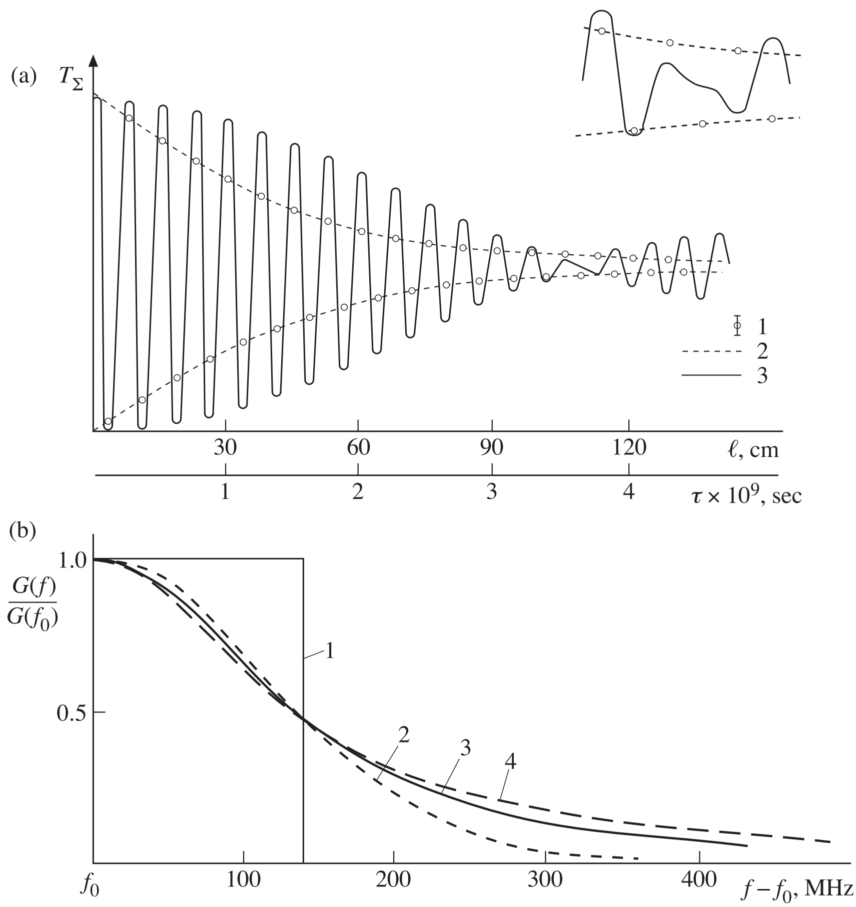

Figure 3.15. Experimental results by Fourier spectroscopy processing. (a) The experimental interferogram. 1: Experimental points (with mean-square errors). 2: The analytic envelope shape (see equation (3.61)). (3) The interferogram of a 'rectangle' passband with $\Delta f_{1/2} = 272\,\text{MHz}$. (b) The amplitude–frequency characteristics (AFC) of the same passband at 3 dB level. 1. The 'rectangle' passband. 2. The Gaussian shape of AFC. 3. The recovered AFC from experimental results. 4. The resonance (Lorentzian) shape of AFC.

rectangular passband and, accordingly, a rounding zero is absent in the experimental interferogram (see insert in Figure 3.15(a)). Its presence is the characteristic indicator of an idealized rectangular passband, which, as we have noted above, cannot be implemented in a physical experiment. The form of AFC studied is at the intermediate position between the Gaussian and Lorentzian forms of passbands (see section 2.5). The values of AFC parameters can be obtained using the relations we already know:

$$q(\tau) = \int_0^\infty G^+(F)\cos(2\pi F\tau)\,\mathrm{d}F, \tag{3.63}$$

and, as a consequence of (3.49), we have

$$4\int_0^\infty q^2(\tau)\,\mathrm{d}\tau = \int_0^\infty [G^+(F)]^2\,\mathrm{d}F. \tag{3.64}$$

Thus, the estimate for the total power passband Δf_{PP} can be found as

$$\Delta f_{\mathrm{PP}} = \frac{\int_0^\infty G^+(F)\,\mathrm{d}F}{G^+(0)} = \frac{q(0)}{2\int_0^\infty q(\tau)\,\mathrm{d}\tau}, \tag{3.65}$$

and for the radiometric passband Δf_{RP} (under the definition given in works by Bunimovitch (1951) and Esepkina *et al.* (1973)) as

$$\Delta f_{\mathrm{RAD}} = \frac{q^2(0)}{2\int_0^\infty q^2(\tau)\,\mathrm{d}\tau}. \tag{3.66}$$

Using the expression for $q(t)$ and the parameters found, we obtain the following values of passbands: $\Delta f_{\mathrm{PP}} = 382 \pm 19$ MHz and $\Delta f_{\mathrm{RP}} = 695 \pm 48$ MHz. The comparison of these quantities with the value of passband $\Delta f_{1/2}$, found from the AFC restoring data, indicates that the power and radiometric bands essentially differ from the $\Delta f_{1/2}$ value due to the power contribution of noise signal components passed through the 'wings' of an amplitude–frequency characteristic, i.e. through that part of the AFC which is not quite reliably determined from reference band measurements (for example, by using sweep-generators).

For more complicated frequency transformations in the amplifying system (for example, in the case of the so-called superheterodyne scheme, see section 3.9) the same form of the Fourier interferogram can have a rather intricate appearance, and the restoring of the total AFC form can represent quite a complicated problem.

Thus, the Fourier spectroscopy method enables us, in the case when a fairly simple and accessible experimental technique is available, to obtain reliable values of radiometric and power bands, as well as to reconstruct at full scale the analytical form of the complicated amplitude-frequency characteristics of amplifying microwave devices.

3.9 BASIC CONCEPTS OF AMPLIFYING DEVICES

Prior to considering radiometric systems with extreme sensitivities, we shall briefly outline the basic concepts of microwave amplifying receivers used in onboard radiothermal and radio-astronomical measurements. Certainly, a vast radio-engineering literature is devoted to these problems. We will be interested here in the qualitative situation only. We make it clear that by a receiving device we mean a purely microwave amplifier (up to the quadratic device). The variety of amplifying devices used can be subdivided, in essence, into three large classes: detector receivers, direct amplification receivers and superheterodyne-type receivers.

3.9.1 Detector receivers

The simplest type of radiometric device is the detector receiver, in which the received microwave fluctuation electromagnetic radiation is immediately directed to the quadratic device, and the input passband of the device is determined by the frequency properties of the antenna system. Such receivers have been applied in radio-astronomy for studying intensive sources (the Sun, for instance), in early onboard radiothermal observations (the investigation of Venus on the *Mariner-II* spacecraft), as well as in receiving systems of the submillimetre band. In these types of receivers, despite high intrinsic noise characteristics, owing to their very broad frequency band (more than 25%), a threshold sensitivity can be achieved (of the order of 2–3 K) which is sufficient for some very important qualitative investigations, for example, for studying the presence of water vapour in the cloudy layer of Venus on the *Mariner-II* spacecraft by microwave techniques, or for searching the signals of extraterrestrial civilizations by microwave techniques in the frequency band of a ‘water hole’. Radiometric instruments of such a type relate to continuous spectrum radiometers and cannot be used for fine investigations of selective emissions.

3.9.2 Direct amplification receivers

Such a reception scheme implies considerable amplification of a signal under investigation in its intrinsic frequency band by means of low-noise receivers. In early radiometers, constructed according to this scheme in 1957–1965 for radio-astronomical purposes, travelling-wave tubes, tunnel-type amplifiers and, later (in 1974–1977), low-noise parametric amplifiers have been utilized. The successes in solid–state electronics made it possible to produce quite low-noise (with a noise temperature of some hundreds of absolute degrees), small-sized and low-power-consuming solid-state amplifiers on the basis of field-effect transistors and tunnel-type amplifiers. The experience of designing and using radiometric onboard complexes has shown that such circuit solutions are worth using now up to frequencies of 50–60 GHz.

3.9.3 Superheterodyne-type receivers

The most widespread receivers of the microwave band are the superheterodyne type, since they are much more sensitive than detector receivers, and, at the same time, are fairly simple in technological implementation. The physical principle of the superheterodyne receiver operation was proposed at the dawn of the development of radio-engineering and has been actively applied in receiving devices of various electromagnetic wavelength bands. The essence of the operating principle is as follows. A fairly powerful electromagnetic harmonic radiation from the internal stable local oscillator is delivered to the nonlinear active element, called a mixer (or a frequency converter), at a frequency close to the working frequency of a signal. In relation to a weak input signal the mixer represents a linear element with harmonically varying active parameters. It can easily be seen that, as a result of such an interaction, we shall have, at the device's output, signals of intermediate and summary frequencies. In this case the whole information load, concluded in the modulation of an input signal amplitude, will be 'transferred' to these frequencies without distortion. A further amplification process occurs at an intermediate frequency by means of intermediate frequency amplifiers (IFA). In this case the signal of summary frequency is suppressed. The value of the intermediate frequency is chosen in the frequency band where there exist low-noise stable amplifiers with the frequency band corresponding to the physical problem under consideration. So, for problems of studying selective emissions this relative (with respect to the input signal frequency) band could be 10^{-5}–10^{-8}, and for problems of studying the continuous spectrum, 10^{-2}–10^{-1}. The development of this scheme is the superheterodyne receiver with a low-noise, high-frequency amplifier at the system's input. Such a scheme makes it possible to obtain in the onboard radiometric systems a record normalized sensitivity of the order of 0.05–0.1 K (see Chapter 14).

The advancement of investigations into the millimetre and submillimetre bands has led to the production of efficient waveguide mixers based on diodes with the Schottky barrier, as well as to using the quasi-optical schemes of mixers with the inclusion of both optical elements (lenses and mirror reflectors), and radiophysical elements (horns, amplifiers). In this case both single and repeated (cascade) frequency transformation is applied.

In concluding this section we notice that progress in designing and producing low-noise receiving systems proceeds with striking dynamics. In the very near future we shall, it seems, witness the production of onboard radiometric receiving systems that use hybrid optical-radiophysical schemes of construction, both for the receiving part directly, and for the information-computation system of onboard radiometric complexes.

3.10 LIMITING SENSITIVITIES OF RADIOMETRIC SYSTEMS

As we have noted above, the limiting sensitivity of radiometric systems is determined by three factors: (a) the sum of the noise temperature of the receiving device and the brightness temperature of the background against which the observations are carried out; (b) the frequency passband of a receiving system; (c) the time constant of the output integrator of a receiving system. Certainly, all these parameters are highly variable, depending on the type of physical problem being investigated and the instrumentation proposed for their performing. In fact, the procedure of optimizing all these parameters for a specific problem is the essence of the observational strategy of an experiment.

Not attempting an analysis of the whole variety of possible physical problems, we shall consider some quantitative evaluations of some of the more contrasting observational situations, which will be useful in the description of particular experiments.

Of course, some of the factors determining the sensitivity of a radiometric system are not completely in the experimenter's control. There is, first of all, the background thermal radiation against which the targeted observations are carried out, as well as the background cosmic radiation (the 'illumination' or 'noises of the firmament', conventionally) and the atmospheric emission. As we shall learn from further considerations, a virtually unremovable radiation comes through the so-called side lobes of antenna systems. If we imagine the possibility of using an ideal (noiseless) amplifier, then in this case the noise temperature in the formula for calculating the threshold sensitivity (3.38) will be determined by the noise temperature of background measurements only. In a real space experiment we have two (apart from the Sun) contrasting versions: the relic background of the universe with a brightness temperature of 2.7 K and 'hot' (certainly, in the sense of thermal radiation) surface covers on the Earth with a brightness temperature of about 300 K (tropical forests, deserts, glaciers). It can easily be understood that some striking results follow from (3.38): with other things (time constant and passband) being equal, the sensitivity thresholds of systems will differ by as much as 100 times (and in investigations of the Sun, 2000 times). Proceeding from this circumstance, the experts (Esepkina *et al.*, 1973; Strukov and Skulachev, 1984, 1986) usually consider reasonable the requirement that the noise of the receiving equipment be not greater than the background noise of the antenna in a particular experiment. Thus, in designing the noise characteristics of radiometric systems it is the radiation the properties of the potential physical objects under study that are taken into consideration in the first place.

The second important factor is the choice and setting of the time constant of an integrator (or the signal accumulation time). Contrast and indicative here may be the values used in the problems of studying the fine features of the radiothermal three-dimensional structure of the relic background of the universe and, for example, in the problems of studying three-dimensional fields of precipitation in the Earth's atmosphere. If in the first type of problems it is necessary to achieve a record sensitivity, of the order of millikelvins (0.001 K), which is caused by the requirements of the physical problem itself (Barreiro, 2000), then for achieving such characteristics the experimenters resort to signal accumulation for about a day ($\tau = 10^4$–10^5 sec) by using a fairly complicated procedure of signal processing (Strukov and Skulachev, 1986).

The other type of problem – the study of the three-dimensional field of precipitation – requires a maximum spatial resolution up to the 'instantaneous field of view', IFOV (one resolution pixel). In this case the surveying of a resolution pixel from a moving vehicle (airplane or satellite) can take insignificant time, of the order of 0.1 sec, for example. Thus, the sensitivity of a radiometric system, already normalized to pixel surveying time (i.e. the sensitivity in a pixel), will 3.3 times worsen as compared to that normalized for 1 sec. It is this value which is often given in the design instrumentation data (Colton and Poe, 1999). However, in studying the three-dimensional characteristics of Earth's surfaces, the necessity very often arises in comparing the radiation properties of quasi-homogeneous extended objects containing thousands and tens of thousands instantaneous pixels.

In this case the space-like brightness sensitivity is considered. This characteristic will be equivalent to signal accumulation in a number of pixels contained in the image. Of course, this characteristic can be much better than the sensitivity within the confines of the resolution pixel.

The third factor determining the threshold sensitivity of a radiometric system is the passband of a high-frequency amplifier. As we have already noted, this parameter is highly determined by the requirements of the statement of the physical problem. So, for studying the fine features of linear spectra (Chapter 11) a frequency resolution in radiometers/spectrometres is required that reaches 10^{-7}–10^{-8} of the central carrier frequency. The values of bands are critical for such observations, since even a small expansion of this band for improving the threshold sensitivity can drastically affect the course of the experiment itself. For studying the radiation characteristics of Earth's surfaces (Chapters 8 and 12) the passband value is considerably less critical, and in continuous spectrum radiometers it can vary within considerable limits, virtually not influencing the final result. It can easily be seen that, other things being equal, the threshold sensitivity of spectroscopic instruments is much worse (almost 10^3 times) than in continuous spectrum radiometers. The problem of achieving sufficient sensitivity for spectroscopic instruments is quite topical now.

So, it is necessary to emphasize once again that the value of a normalized threshold sensitivity in itself, certainly characterizes the radiophysical properties of an instrument as a whole, but still does not determine, however, the efficiency of using a radiometric device in any remote sensing problem. From the observational practice viewpoint, of importance are also signal accumulation parameters and the passband (the amplitude–frequency characteristic) and the relationship between them. The ultimate efficiency of a remote sensing instrument can be determined only after detailed joint analysis of instrumentation parameters and spatial-temporal and radiation properties of the physical object under study. In the overwhelming majority of cases the onboard equipment developers have to search for compromise solutions in the construction of onboard radiometric complexes. We shall consider appropriate examples in analysing various types of microwave sensing problems in subsequent chapters (Chapters 8, 11, 12 and 14).