4

Thermal fluctuations and their fundamental laws

The subject of investigation in this chapter is the fundamental law of nature that relates the quantum fluctuation radiation of an object of any physical nature with its dissipative properties in macroscopic scales and is called the fluctuation–dissipation theorem – FDT. Attention is chiefly given to the physical aspect of the problem. Two approaches, which are important for remote sensing and instrumental applications, are analysed in this chapter. They are the quasi-stationary FDT approximation, called the Nyquist formula, and the geometric-optical approximation, the Kirchhoff law. In addition, methodological issues of the application of FDT results under real remote sensing conditions are considered.

4.1 THERMAL RADIATION AND THERMAL FLUCTUATIONS: A HISTORICAL REVIEW

One of the fundamental factors that explains the principal significance of thermal radiation (sometimes called thermal electric fluctuations) in remote sensing and astrophysical applications, is the fairly transparent physical relationship between the recorded radiation and the internal thermal structure of a physical object and its physical-chemical and physical-geometric features. In fact, all fundamental results of both remote sensing (of the Earth and planets) and astrophysics obtained up to the present, are largely based on using the results of remote observation of thermal radiation (thermal fluctuations), which is generated and reveals itself (certainly, in the observational respect) in different parts of the electromagnetic spectrum, depending on its temperature and physical properties. Of course, in addition to thermal radiation, many other electromagnetic emissions either fall from space to the Earth or are formed directly under Earth conditions. These emissions also have a fluctuation character, but are not pertinent to thermal radiation physics. The separation or extraction of various types of emissions from the experimental data sometimes represents a complicated scientific problem in itself. In this book, as mentioned above, we shall consider thermal radiation issues only.

All physical objects having physical temperatures other than absolute zero are continuously emitting a fluctuating electromagnetic field arising from internal energy, which stipulates the possibility of spontaneous transitions between vibration–rotation levels of molecules in gases, oscillations of molecules in liquid and solid bodies and oscillations of a lattice in solid bodies, with subsequent de-excitation of electromagnetic quanta. The radiation has a typically quantum character and cannot be described within the framework of the classical Maxwellian theory of electromagnetism. The radiation energy covers a very broad range of wavelengths and has (as is usually stated in radiophysics and optics) a continuous spectrum of rather complicated form, the position of maximum of which depends on the thermal temperature of matter. As to terminological approaches, the literature offers a spectrum of names for this radiation: emission, emitted radiation, Planck's emission, black-body radiation, thermal emission, radiothermal emission, radio-emission, grey-body radiation, outgoing radiation.

The study of various parts of the electromagnetic spectrum of thermal radiation, and thermal fluctuations in general, has proceeded rather non-uniformly in the historical respect (Rytov, 1953, 1966; Levin and Rytov, 1967; Schopf, 1978). In studying thermal electrical fluctuations and thermal radiation there are two ranges of questions, the relation between which was elucidated long ago, but only in the early 1950s was it formulated mathematically as a unified theory called the fluctuation–dissipation theorem.

One of the areas we speak about arose considerably earlier in time; it concerns the issues of the thermal radiation of heated bodies considered in the optical wave-length band. Researchers have been interested in the relation between the emitting body and the environment from the beginning of the nineteenth century (P. Prevost, B. Stewart, A. J. Angstrom). But only G. R. Kirchhoff had sufficient insight into the subject to elucidate the primary ideas concerning 'ray-radiation' (emission) and absorption. Kirchhoff's work has rested upon a discovery made some months before that event by Kirchhoff and Bunsen: they found that Fraunhofer's absorption lines in the solar spectrum coincided with the lines of emission of known vapours and gases. Kirchhoff himself evaluated his discovery as the proof of the fact that matter outside the Earth consists of known chemical elements. Doubtless, this was one of the first outstanding discoveries in astrophysics (see Schopf (1978) for more details).

One of the fundamental results obtained by Kirchhoff, on the basis of application of thermodynamic laws to equilibrium thermal radiation, was the proof of the fact that the spectral density of this radiation is a universal function of frequency and temperature. The complete determination of the form of the universal function constituted the problem of the next stage of development in thermal radiation theory. The final solution of this problem, based on the quantum hypothesis and resulting in the expression for spectral density of the equilibrium (absolutely black-body) radiation, that is valid for any frequency, was given by M. Planck. The detailed and fascinating presentation of the (sometimes dramatic) history of this discovery was given in a book by H.-G. Schopf (1978).

Another, no less important, result which is very significant for the practice of remote sensing and astrophysical investigations was the theoretical proof, in the geometric-optical approximation, of the law called the Kirchhoff law. This law states that the ratio between 'emissive' ability (or radiation intensity) and absorbing ability is identical for all bodies (irrespective of their shape, chemical composition, aggregate state, surface properties, etc.) for a given temperature and for a given frequency. Subsequent investigations have shown that the universal constant in the Kirchhoff law is closely related to the spectral intensity of equilibrium radiation inside the enclosure of a thermostat. At present, several forms of the Kirchhoff law presentation are used in the theory and practice of remote sensing and astrophysical investigations, the physical sense of which is, certainly, identical. Some of these forms will be described in Chapter 6.

Another area where researchers have again (though much later) encountered thermal fluctuations is the so-called 'noise' in electrical circuits and, first of all, in amplifying devices, whose noise properties have already been mentioned many times in this book (see Chapter 3). The close relation between electrical noises and thermal radiation lies in the fact that this radiation represents a wave electromagnetic field generated by thermal electrical fluctuations in physical bodies of various natures. The physical explanation of the fact that a unified and rather general theoretical approach to such closely related physical phenomena has been absent for a long time lies in the great distinction between the frequencies of the electromagnetic oscillations of interest in each of the aforementioned areas. The questions related to thermal radiation arose and have been studied as optical problems using the methods of geometric optics (see section 1.6). On the other hand, electrical noise was found experimentally in the band of low radio frequencies, which made it possible to consider them within the framework of the theory of quasi-stationary currents only (see section 1.6).

However, in the 1940s the intensive development of radar engineering gave rise to considerable growth in the sensitivity of radio and radar receiving equipment (Skolnik, 1980; Brown, 1999). This made it possible to reliably record, in the decimetre and centimetre bands, thermal electromagnetic radiation coming from natural physical objects situated both on the Earth's surface, and in space. It was this technological basis on which the new science – radio-astronomy – arose and continues to progress actively now (Esepkina *et al.*, 1973; Ruf, 1999; Barreiro, 2000; Kardashiov, 2000). A little later aerospace radio thermal location (microwave radiometry) and scatterometry of the Earth surface arrived and continue to be efficiently developed (Basharinov *et al.*, 1974; Sharkov and Etkin, 1976; Bass *et al.*, 1977; Raney, 1983; Kalmykov, 1996; Carver *et al.*, 1985; Shutko, 1986; Massonnet, 1996). Thus, the areas of thermal radiation and electrical noise 'have touched each other' closely in the microwave band.

Though the existence of electrical fluctuations of thermal origin in radio-engineering circuits and receivers has been obvious since the first steps in the development of Brownian motion theory in statistical physics at the beginning of the twentieth century, their experimental detection became possible as a result of the improvement of radio engineering devices and, first of all, amplifying systems at the end of the 1920s. In 1927 J. B. Johnson found that at an output of the amplifier, to the input of which the active resistance is connected, additional noise – the chaotic voltage – was observed. As was found out later, this noise is of purely Gaussian type, and its intensity (the mean square – the variance) grows linearly with resistance R at an input and with increasing physical temperature. Almost simultaneously with these experiments H. Nyquist, using the existing physical concept of a random electromotive force (emf), localized in the active circuit, has showed that the spectral intensity (Wiener's spectrum), $G^{+}(f)$, of the fluctuation emf, localized in the arbitrary passive two-terminal circuit with impedance $\dot{Z}(\mathrm{j}2\pi f)$, is

$$G^{+}(f) = 4kT\,\mathrm{Re}\,\dot{Z}(\mathrm{j}2\pi f), \tag{4.1}$$

where k is the Boltzmann constant (see Appendix A), and T is the absolute temperature. In such a form, this formula, called the Nyquist formula (or the Nyquist theorem), gives the spectral intensity in the unit interval of positive frequencies and is valid in the non-quantum region of frequencies and temperatures, i.e. for $hf \ll kT$ (here h is Planck's constant, see Appendix A). The rigorous quantum-mechanical generalization of this formula, whose necessity was still pointed out by Nyquist himself, was performed much later, however, as a result of the quantum-mechanical derivation of the fluctuation–dissipation theorem (see, for instance, Levin and Rytov, 1967). The complete form of the spectral density, which is valid both for low temperatures, and for sufficiently high frequencies, $hf > kT$, is as follows:

$$G^{+}(f) = 2hf \coth\left(\frac{hf}{2kT}\right) \mathrm{Re}\,\dot{Z}(\mathrm{j}2\pi f) \tag{4.2}$$

where $\coth x = (\exp(2x)+1)(\exp(2x)-1)$ is the hyperbolic cotangent. From quantum mechanics we know the expression for the mean energy of the so-called quantum oscillator:

$$\Theta(\omega, T) = \Theta(f, T) = \frac{hf}{2} + \frac{hf}{\exp(hf/kT) - 1} = \frac{hf}{2}\coth\left(\frac{hf}{2kT}\right), \tag{4.3}$$

in this case

$$hf = \hbar\omega = \frac{h}{2\pi}\omega.$$

In such a case the Nyquist formula can have the more compact quantum form:

$$G^{+}(f) = 4\Theta(f, T)\,\mathrm{Re}\,\dot{Z}(\mathrm{j}2\pi f). \tag{4.4}$$

The further development of the theory of thermal fluctuations resulted in the appearance of a set of derivations of this formula and in far-reaching generalizations, from which it issues as a very simple special case. First of all, we should point out here the transition from concentrated fluctuation forces to detached random fields (both electrical and magnetic, in the general case) and the construction of spatial correlation functions for spectral amplitudes of detached fields in the frequency bands not limited by the quasi-static condition (see section 1.6) (Rytov, 1953). At that time, in the early fifties, H. B. Callen with co-workers proved the fluctuation–dissipation theorem (FDT), which generalized Nyquist's result: first, to dissipative systems of an arbitrary physical nature; second, to the quantum region of frequencies and temperatures; and, third, to thermodynamic fluctuations described by any number of discrete functions of time. Thus, the possibility of regular application of FDT to distributed physical systems was opened up. This is just what was done a little later when it was applied to the Maxwell equations (Landau and Lifshitz, 1957), and the general FDT formulation was established for the case of distributed dissipative systems (Levin and Rytov, 1967).

Thus, as a result of the 'merging' of the two aforementioned directions, the theory of thermal fluctuations (thermal radiation) in electrodynamics represents one of the most important applications of the general theory of thermal fluctuations to arbitrary macroscopic systems. First of all, we shall indicate the principal importance of using this theory in the observational practice of microwave remote sensing, namely in those cases where the size of physical bodies is of the same order as the working wavelength (see section 1.6), and the diffraction phenomena in interaction problems make a noticeable and, sometimes (as in problems of emission and scattering from the wavy sea surface), overwhelming contribution.

Prior to considering the results of the application of the theory of thermal fluctuations to remote sensing, we shall discuss briefly, and mainly at the qualitative level, the physical essence of the fluctuation–dissipation theorem.

4.2 THE FLUCTUATION–DISSIPATION THEOREM: A QUALITATIVE APPROACH

The fluctuation–dissipation theorem is one of the fundamental laws of statistical physics. It establishes for an arbitrary dissipative physical system the relationship between the spectral density of spontaneous equilibrium fluctuations and its non-equilibrium properties, the energy dissipation in a system in particular. The detailed quantum-mechanical derivation of this theorem can be found both in original works by H. B. Callen with co-authors (Callen and Welton, 1951; Callen and Green, 1952), and in a series of textbooks and monographs on statistical physics (Landau and Lifshitz, 1957; Rytov, 1953, 1966; Levin and Rytov, 1967).

To elucidate the qualitative physical issues, it is sufficient to consider the special case where the fluctuations in a system are determined by a single random quantity. We designate it by ξ and assume that at the equilibrium state its mean value is zero. Suppose also, that the system is situated in a thermostat and, accordingly, obeys the canonical Gibbs (J. W. Gibbs) distribution. Further, the interaction with a thermostat is supposed to be weak, so the system's energy can be introduced, which is uniquely determined by the state of the system itself. Using quantum-mechanical approaches, it can be shown that the spectral density of equilibrium fluctuations of the quantity ξ is expressed in terms of the levels of energy E_n of the considered system and magnitudes of matrix elements $\xi_{n,m}$. However, the actual calculation of E_n and $\xi_{n,m}$ for a real macroscopic system requires consideration of the micromechanism of fluctuations, and, generally, for real physical bodies the problem seems to be virtually hopeless. Note that the intensity of spontaneous internal thermal fluctuations is described by means of E_n and $\xi_{n,m}$. Certainly, the macroscopic dynamics of a system is not reflected at all in any way in the quantum-mechanical structure, since the macroscopic process in a system, by which we mean the average variation of system's parameters $\bar{\xi}$, may be caused in a dissipative system by the external effect of macroscopic forces only.

The theoretical and practical value of FDT consists, in particular, in the fact that for thermodynamic equilibrium systems it gets rid of the necessity to find directly E_n and $\xi_{n,m}$ for a real physical body by expressing the spectral density of fluctuations in terms of a particular macroscopic characteristic of a system – its general susceptibility.

Suppose the system under consideration is disturbed by the effect of the external force $f(t)$. Now let this force be sufficiently small that the macroscopic response can be found from the linearized equation of motion and, accordingly, the spectral amplitude $\bar{\xi}$ to be linearly related with the spectral amplitude of the disturbing force $f(\omega)$:

$$\overline{\xi(\omega)} = \dot{\alpha}(\mathrm{j}\omega) f(\omega). \tag{4.5}$$

Quantity $\dot{\alpha}(\mathrm{j}\omega) = \alpha'(\omega) - \mathrm{j}\alpha''(\omega)$, determined by relation (4.5) and called the general susceptibility, is introduced for weak effects allowing for linearization of the macroscopic equations of motion for the physical system under consideration. Since for real $f(t)$ quantity $\bar{\xi}(t)$ should also be real, we have $\dot{\alpha}(-\mathrm{j}\omega) = \dot{\alpha}(\mathrm{j}\omega)$, i.e. $\alpha'(\omega)$ is an even function, and $\alpha''(\omega)$ an odd one.

Often in physical practice the basic equation for a system is used in the form of macroscopic response for the rate of time variation of the basic parameter of a system, and then the relation for spectral amplitudes of velocity $\bar{\dot{\xi}}(t)$ and external force $f(t)$ is presented as:

$$\overline{\dot{\xi}(\omega)} = \dot{Y}(\mathrm{j}\omega) f(\omega), \tag{4.6}$$

where coefficient $\dot{Y}(\mathrm{j}\omega)$ is called the admittance of a system. The general susceptibility $\dot{\alpha}(\mathrm{j}\omega)$ is associated with the admittance of a system by relation $\dot{Y}(\mathrm{j}\omega) = \mathrm{j}\omega\dot{\alpha}(\mathrm{j}\omega)$ (see Appendix B, equation (B.7)). The reciprocal quantity to the admittance is called the impedance of the system, $\dot{Z}(\mathrm{j}\omega) = 1/\dot{Y}(\mathrm{j}\omega)$. If we address the theory of electrical circuits (see sections 1.6 and 2.6), then we find that similar parameters are introduced for describing the processes in electrical circuits as well.

The quantum-mechanical consideration of energy dissipation in a system eventually results in the following important relation between the spectral density of fluctuations, $G_\xi(\omega)$, and the general susceptibility, which just expresses the physical essence of FDT:

$$G_\xi(\omega) = \frac{\Theta(\omega, T)}{\pi\omega}\,\alpha''(\omega). \tag{4.7}$$

Here $\Theta(\omega, T)$ is the mean energy of a quantum oscillator (4.3); in this case the frequencies are considered in the whole frequency band (both positive and negative).

The fundamental relation (4.7) indicates that the spectral intensity of equilibrium fluctuations is determined by the imaginary part of a system's susceptibility, that is the quantity describing the dynamic behaviour of a linearized macroscopic system beyond any relation with the fluctuations. Note here the following important point: the dynamics of a system under strong external effects, where the macroscopic equations of motion can also be nonlinear, has no relation to thermodynamic fluctuations. So, the macroscopic non-linearity (the deviation from Ohm's law in electrical circuits) can be revealed only for essentially non-equilibrium distribution of current carriers in conductors, or, in other words, beyond the FDT action framework (Levin and Rytov, 1967).

According to (4.7), the total intensity (the variance) of fluctuations in a system is:

$$\overline{\xi^2} = \int_{-\infty}^{\infty} G_\xi(\omega)\,\mathrm{d}\omega = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\Theta(\omega, T)}{\omega}\alpha''(\omega)\,\mathrm{d}\omega. \tag{4.8}$$

Let us consider now another form of FDT, which is fairly often utilized. It is based on the so-called Langevinian conception of fluctuation forces. (It was this conception that was utilized by Nyquist for deriving his formula (4.1).) These equivalent random forces are introduced into linearized macroscopic equations of motion of a system as a 'reason' for fluctuations, i.e. these forces are introduced along with true external forces. Thus, by quantity $f(\omega)$ in equation (4.5) can be meant the spectral amplitude of a random equivalent force, and equation (4.5) can be understood as the equation relating spectral amplitudes of macroscopic random processes $\xi(t)$ and $f(t)$. Recalling the relation between the spectra of linearly bound processes (2.65), we obtain from relations (4.5) and (4.7) the formula for the spectral intensity G^f of the fluctuation force $f(t)$:

$$G^f(\omega) = \frac{G_\xi(\omega)}{|\dot{\alpha}(\mathrm{j}\omega)|^2} = \frac{\Theta(\omega, T}{\pi\omega}\frac{\alpha''(\omega)}{|\dot{\alpha}(\mathrm{j}\omega)|^2}. \tag{4.9}$$

As an example, we shall use this formula for deriving the Nyquist relation. As we noted above (section 1.6), in the general case for linear concentrated circuits the generalized coordinates are the charges q_j and the generalized velocities are the currents $\dot{q}_j = I_j$. The spectral amplitudes of currents I_j and electromotive forces $\mathscr{E}_j$ are specified by Kirchhoff's generalized equations (Krug, 1936; Rytov, 1966)

$$\begin{aligned} T_j &= \sum_k \dot{Y}_{jk}(\mathrm{j}\omega)\,\mathscr{E}_k \\ \mathscr{E}_j &= \sum_k \dot{Z}_{jk}(\mathrm{j}\omega)\,I_k, \end{aligned}, \tag{4.10}$$

here $\dot{Y}_{jk}(\mathrm{j}\omega)$ and $\dot{Z}_{jk}(\mathrm{j}\omega)$ are mutually reciprocal matrices of the admittance and impedance k of electrical circuits. If we deal with a single active resistance R, then its admittance is $1/R$ and, accordingly, the general susceptibility is $\dot{\alpha}(\mathrm{j}\omega) = 1/\mathrm{j}\omega R$, and its imaginary part is $\alpha''(\omega) = 1/\omega R$. Thus, the intensity of fluctuations according to relation (4.9) will be equal to:

$$\sigma^2 = \overline{\mathscr{E}^2} = \frac{2}{\pi}R\int_0^{\infty} \Theta(\omega, T)\,\mathrm{d}\omega. \tag{4.11}$$

In the classical approximation, i.e. when $\hbar\omega \ll kT$ and, here, $\Theta(\omega, T) = kT$, and using the Langevinian conception (4.9), we (transferring to positive frequencies) obtain the well-known Nyquist relation we have already used many times before:

$$\sigma^2 = \overline{\mathscr{E}^2} = 4kTR\,\Delta f. \tag{4.12}$$

In studying the limiting sensitivity of mechanical systems, such as mechanical gravitational wave detectors (Yamamoto *et al.*, 2001), the same Nyquist formula is used in a slightly different form (certainly, without changing the physical essence of the phenomenon), namely, in the form of the relationship between the thermal noise spectrum and the mechanical response of a system:

$$G(\omega) = -\frac{4kT}{\omega}\,\mathrm{Im}\,\dot{H}(\mathrm{j}\omega). \tag{4.13}$$

The transfer function, $\dot{H}(\mathrm{j}\omega)$, is written as

$$\dot{H}(\mathrm{j}\omega) = \frac{\dot{X}(\mathrm{j}\omega)}{\dot{F}(\mathrm{j}\omega)}, \tag{4.14}$$

where $\dot{F}(\mathrm{j}\omega)$ and $\dot{X}(\mathrm{j}\omega)$ are the Fourier components of the applied mechanical force and the displacement at the observation point, respectively. The imaginary part of the transfer function represents the phase lag between the force and the displacement, which is related to the dissipation of a system.

Below we shall summarize the basic qualitative components of FDT.

(1) Any dissipative system of arbitrary physical nature possesses spontaneous equilibrium fluctuations whose intensity is determined by the macroscopic dissipative properties of a system. As examples of various FDT applications to concentrated and distributed systems, we point out the investigations of thermal fluctuations in liquids (Landau and Lifshitz, 1957), in mechanical systems, in plasma, in electronic gas, in hydrodynamics and, which is closest to our subject, the studies of thermal fluctuations of electromagnetic fields (Levin and Rytov, 1967).
(2) The FDT action spreads to any relationship between frequencies and temperatures, beginning with the classical limit $hf \ll kT$, both for low temperatures and for high frequencies, $hf \gg kT$.
(3) In applying FDT to the electrodynamics, its action spreads to any relationship between the geometrical size of a system and the working wavelengths of the fluctuation electromagnetic field of radiation. In the case of geometrical optics, $L \gg \lambda$, FDT asymptotically 'transfers' into the form of Kirchhoff's law, and in the quasi-static case, $L \ll \lambda$, into the Nyquist formula (4.1). Surprising is the fact that in the intermediate, most complicated (diffraction) case, $L \sim \lambda$, it was possible to find in the most general case (Rytov, 1966; Levin and Rytov, 1967) a rather transparent relationship between the emissive and absorbing properties of media (Rytov's formulae) (see section 4.3).

4.3 THERMAL FLUCTUATIONS IN THE ELECTRODYNAMICS

As we have already indicated, the most important application of the general theory of thermal fluctuations in arbitrary macroscopic systems is the theory of thermal fluctuations (radiation) in Maxwell's electrodynamics, as applied, first of all, to microwave sensing problems. There are two important aspects here.

The first aspect is associated with the fact, that the general conditions of the macroscopic electrodynamics applicability will be spread to the similar approach (Stratton, 1941; Landau and Lifshitz, 1957; Levin and Rytov, 1967). First, it is necessary that the inhomogeneities of macrofields (the working wavelength of the electromagnetic radiation) be much larger than the microinhomogeneities caused by the molecular structure of emitting bodies. This requirement is fulfilled for a broad range of electromagnetic radiation, including the band of optical frequencies. Besides, the phenomenological concept of matter as a dielectric continuum (dielectric formalism) (section 1.6) in Maxwellian theory implies the exclusion from the statistical electronics, i.e. from accounting for such parameters as the elementary charge, the number of elementary charges per unit volume, thermal velocities of microcharges, their free path length, etc. Nevertheless, since the electrodynamics part of a problem is solved in this case by means of the general Maxwell equations, the results obtained cover all diffraction phenomena occurring under the given physical and geometrical conditions, including, naturally, the extreme cases as well, for instance, the quasi-stationary approximation and the geometrical optics.

The second aspect concerns the following circumstance. As we have noted above, thermal radiation has a typically quantum character and cannot be straightforwardly described within the framework of the classical Maxwellian theory of electromagnetism. Within the phenomenological theory framework the fluctuation electromagnetic field is represented as the field generated by random 'detached' sources of Langevinian type, distributed in the volume of a medium under investigation (Landau and Lifshitz, 1957; Levin and Rytov, 1967). In spite of using a rather artificial approach – the introduction of detached fluctuation fields – such an approach allows us to formulate any problem on equilibrium thermal fluctuations of electromagnetic quantities as a usual boundary value problem of electrodynamics and, thereby, to use in thermal radiation problems the full power of diffraction electrodynamics. And most striking is the fact that the strict (diffraction) theory of fluctuation fields in electrodynamics can be reduced, in the most general form, to a simple and delicate form of relationship between fluctuation (radiative) and dissipative characteristics of physical media (Levin and Rytov, 1967). Following the aforementioned authors, we shall first consider the contents of the electrodynamic FDT as applied to the electromagnetic field and then the diffraction generalization of the Kirchhoff law.

So, as we have already noted, within the phenomenological theory framework the fluctuation electromagnetic field can be considered as the field generated by random detached currents spread in a medium. To calculate the energy characteristics of the fluctuation field, including spatial characteristics of the fluctuation illuminated radiation, it is necessary to know the spatial correlation of the spectral amplitudes of these random currents, which, strictly speaking, constitutes the matter of the electrodynamic FDT. Both in distributed and discrete systems, the FDT allows us to associate the correlation functions of detached fields (currents) with the dissipative properties of a system, which are laid down in macroscopic (linearized) equations of the system. If these equations are Maxwellian (see section 1.6), then they just determine the spatial correlation of detached electrical and magnetic fields, and in this case the dissipative properties of a medium will be described by macroscopic constitutive equations of the medium (see section 1.6).

In applying the general theory of thermal fluctuations to the electromagnetic field the conventional form of field equations is utilized (see section 1.6 and equation (1.1)). In this case the conductivity current and free charges are not separated from the polarization current and polarization (i.e. in relations (1.1) quantities j and ρ are supposed to be zero). As far as the fluctuation 'forces' are concerned, they can be expressed in different ways, either as detached inductions, or as detached strengths, or as detached currents. If we make use of the last approach, then the macroscopic equations of the electromagnetic field, to which FDT should be applied, are represented by the Maxwell equations of the form (in the Gaussian system of units):

$$\begin{aligned} \operatorname{rot} \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c} \mathbf{j}_{\mathrm{e}} \\ \operatorname{rot} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} - \frac{4\pi}{c} \mathbf{j}_{\mathrm{m}} \end{aligned}, \tag{4.15}$$

where $\mathbf{j}_{\mathrm{e}}$ and $\mathbf{j}_{\mathrm{m}}$ are the detached fluctuation currents (electrical and magnetic), which 'cause' thermal fluctuations of all electrodynamic quantities. Many authors have utilized various physical approaches in applying FDT to the electromagnetic theory: the use of the discrete and continuous FDT forms, and the use of the detailed equilibrium principle, as well as a number of indirect physical considerations. The use of various physical approaches results in the following expression for spatial spectral amplitudes of random currents for the isotropic medium:

$$\overline{\mathbf{j}_{ej}(\mathbf{r}) \cdot \mathbf{j}^{*}_{ek}(\mathbf{r}')} = -\frac{j\omega\Theta(\omega, T)}{8\pi^2} [\dot{\varepsilon}^{*}(\mathbf{r}) - \dot{\varepsilon}(\mathbf{r}')] \, \delta_{jk} \, \delta(\mathbf{r} - \mathbf{r}'), \tag{4.16}$$

where subscripts j and k denote spatial components $(j = k = 1, 2, 3)$. Recalling the expressions for the dielectric constant of a medium, this expression can be simplified and reduced to the form:

$$\overline{|\mathbf{j}_e(\mathbf{r})|^2} = \frac{\omega\Theta(\omega, T)}{4\pi^2} \varepsilon''(\mathbf{r}). \tag{4.17}$$

Expressions (4.16) and (4.17) just constitute the matter of the electrodynamic form of FDT, generalized to continuous dissipative systems, in application to the electromagnetic field. The physical essence of these expressions corresponds to the basic matter of FDT, namely: the intensity of electromagnetic fluctuations (electromagnetic radiation) in a medium is immediately directly associated with macroscopic dissipation properties in a system, which are reflected by constitutive equations of a medium (see section 1.6). Of course, all spatial features of the medium under study, reflected in constitutive equations (the inhomogeneity, anisotropy), will directly stipulate the correlation properties of intensity of detached sources and, certainly, the properties of the field of thermal radiation of a studied physical object, which just will be recorded by an external observer. The developed approaches are valid for the homogeneous medium only under the condition of retaining the prerequisites for the phenomenological description of the medium in Maxwellian electromagnetism. In other words, in this case the medium can be partitioned into physically elementary volumes, which are small when compared with macroscopic inhomogeneities of a medium, but contain a great number of microparticles. Under these conditions the statistical approach to the description of the state of a medium in such elementary volumes is conserved. If the conditions of distribution of microparticles in these volumes are close to equilibrium (see section 4.4) with some local temperature, then formulas (4.16) and (4.17) can also be spread to a non-uniformly heated (none-quilibrium) medium, provided that the temperature in the coefficient $\Theta(\omega, T)$ is a function of a point.

However, from the viewpoint of observational remote sensing practice, the expressions obtained are not sufficient, since they determine the state inside the medium, whereas remote sensing instruments record the electromagnetic radiation escaped from the medium into free space, where the remote-sensing instruments are situated. Since the detached currents are distributed over the whole volume of an emitting body, the obvious method of calculating the external ('illuminated') electromagnetic field consists in using the regular methods of electrodynamics for the region determined by the emitting body's form. However, because we deal with the spatial distribution of detached currents in the medium volume, for the overwhelming number of real media (or physical bodies) the formulation of a complete electrodynamic problem can be very complicated. As we have noted above (section 4.1), the intensities of the field emitted by the medium (which are of interest for us from the remote sensing point of view) can be found by the more simple and physically transparent method developed by Levin and Rytov (1967).

The essence of the method is as follows. The two fields are compared: the radiation field, which is recorded by an instrument in the external (with respect to the medium studied) space, and the supplementary field of the planar wave coming in the direction in which the radiation intensity is of interest for ourselves. Those diffraction fields should be taken as supplementary, which are formed at irradiating the investigated medium by waves issuing from elementary dipoles situated at the outer space points we are interested in. The application of the electrodynamic theorem of reciprocity in combination with FDT results in the universal relations between the spatial correlation functions of spectral amplitudes of the radiation field of the given medium, on the one hand, and the thermal losses (in this field) of the diffraction supplementary field generated by the dipole situated at the observation point. The considered approach is no longer bound by any limitations between the characteristic body size, L, and the working wavelength, λ (as in the case of the

Kirchhoff law and the Nyquist formula), and allows us to find any spatial characteristics of the radiation field at any distance from a body.

If by $\mathbf{A}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ are meant any two of six components of strengths $\mathbf{E}$, $\mathbf{H}$ of the thermal radiation field, considered at two different distances $\mathbf{r}_1$ and $\mathbf{r}_2$ from a body, then the mean value of bilinear combinations of components of strengths of the radiation field is related with thermal losses of the total diffraction field from point sources $Q_{AB}(\mathbf{r}_1, \mathbf{r}_2)$ as follows:

$$\mathbf{A}(\mathbf{r}_1) \cdot \mathbf{B}^*(\mathbf{r}_2) = \frac{2}{\pi} \Theta(\omega, T)\, Q_{AB}(\mathbf{r}_1, \mathbf{r}_2). \tag{4.18}$$

And if the question is the intensity of radiation of the whole body volume, considered at one point of the outer space and having specific polarization, then (4.18) can be transformed to the integral form:

$$\overline{|E_{\mathrm{p}}(\mathbf{r})|^2} = \frac{2}{\pi} \int_V \Theta[\omega, T(\mathbf{r}_1)]\, \mathrm{d}\mathbf{Q}_{EE}(\mathbf{r}, \mathbf{r}_1). \tag{4.19}$$

where the integral is taken over the whole volume, V, of the emitting body (a medium) with regard to the field of temperature $T(\mathbf{r}_1)$ non-uniformly distributed inside a body. Here $E_{\mathrm{p}}(\mathbf{r})$ is the projection of the fluctuation electrical vector at the observation point P of the outer space with current radius-vector $\mathbf{r}$ on the direction of the dipole moment, and $\mathbf{r}_1$ is the current radius-vector, determining the point of position of the detached current inside the emitting body. The solution contains both a wave (far) field, carrying the energy away from a body, and a quasi-stationary (near) thermal field, which is concentrated in a layer adjacent to the body surface, whose thickness is of the order of the working wavelength, and rapidly decreases with the distance from a body. The quasi-stationary fields do not participate in the energy transfer (see Chapter 5), but make their contribution to the volume density of energy of the fluctuation field, which sharply grows near the emitting body surface. The diffraction effects, recorded in the far field, as well as the detection and calculation of the quasi-stationary field, represent principally new advantages of the fluctuation electrodynamics as compared to the classical theory of thermal radiation (the Kirchhoff laws).

Formulae (4.18) and (4.19) are, in essence, basic equations to the whole theory of thermal electromagnetic fields. These formulae (sometimes called Rytov's formulae), which relate the second moments of spectral amplitudes of the fluctuation field with thermal losses of the diffraction field of point sources, can be considered as the generalization of the classical expression of the Kirchhoff law to the diffraction region (Rytov, 1966; Levin and Rytov, 1967).

Certainly, to find the diffraction field losses it is necessary, again, to solve the appropriate electrodynamic problems by regular methods. However, these problems are much simpler, than those considered above, where it was necessary to solve the problem on spatially distributed detached currents, i.e. the currents distributed in a complicated manner in the emitting body's volume. In some cases it is possible to use for this purpose either the existing solutions of diffraction problems or the approximate solutions, where some features of a specific problem are used (such as the local application of geometric optics, the presence of a skin effect, the local scales of surface roughness, etc.).

As an example, we shall consider the results of the solution of the aforementioned problem for the situation, which is often encountered in remote sensing practice. We mean the case where it is necessary to measure the intensity of radiation of the absorbing half-space with a smooth boundary with a remote sensing device standing outside of this space. Such a model situation is a basic one in analysing any experimental data, obtained in sounding the Earth's surface, and for this reason we shall repeatedly return to these results throughout this book. Let the half-space $z < 0$ be filled with an isotropic conducting medium with a complex dielectric constant, and in the region $z > 0$ the medium is isotropic too, but it is transparent with the real index of refraction, n (see section 1.6). For the wave field, i.e. the radiation field, which can be recorded by the external (with respect to the emitting medium) instrument, the solution of the fluctuation electrodynamic problem should result immediately in the Kirchhoff law. In addition, the solution will also contain the quasi-stationary field components, which, however, very rapidly decrease with the distance from the surface and make no contribution to the energy flux. The experimental recording of such a field is a rather complicated and ambiguous problem. In accordance with the developed methodology (Levin and Rytov, 1967), the electrical and magnetic dipoles with corresponding dipole moments are placed at any point of the transparent half-space (over the planar boundary), and then it is necessary to find the diffraction field losses in the emitting half-space. This problem, called Zommerfeld's problem, is a classic one in the problem of radio wave propagation over the Earth's surface (Stratton, 1941; Alpert *et al.*, 1953).

The complete solution of this problem leads to the following result: the power characteristics of the fluctuation thermal field in the far region (the radiation zone), the Poynting vector in particular, do not depend on the distance from the medium and can be expressed, for a fixed direction and fixed body angle, as the radiation intensity (see Chapter 5 for more details) as follows:

$$I_\omega = I_{\omega 0}\, n^2 (1 - |R|^2), \tag{4.20}$$

where by $I_{\omega 0}$ is meant the equilibrium intensity formed inside the emitting body, R is the Fresnel coefficient of reflection of a planar electromagnetic wave from the smooth boundary of the emitting medium (with account taken of polarization and the angle of observation), and n is the index of refraction of a transparent external medium. The expression presented is, in essence, the Kirchhoff law for radiation of the absorbing half-space. The physical sense of this relation is fairly transparent: the formed equilibrium intensity of an infinite half-space undergoes reflection at the planar interface boundary. In this case the value of the energy, reflected inside the medium, will be equal to $I_{\omega 0}\,|R|^2$. Thus, the energy will be illuminated into the outer space and recorded by a remote instrument in accordance with relation (4.20).

4.4 LOCAL THERMAL EQUILIBRIUM

As we have already noted, FDT is valid for systems with thermodynamic equilibrium. In statistical physics by thermal equilibrium is meant the physical state, into which any closed macroscopic system comes after a fairly long time interval has elapsed. At thermodynamic equilibrium the detailed balance is established; that is, any elementary process in a system is balanced by the corresponding reverse process. The detailed balance takes place for the processes, which change the kinetic energy and the direction of motion of both the macroscopic particles of a system and the state of elementary particles, atoms, molecules and ions, and the state of their excitation for the processes of ionization and recombination, dissociation and formation of molecules, etc. At the thermal equilibrium state the parameters of a system do not change in time; however, they can undergo thermal fluctuations about their mean values. The thermal radiation arises under the detailed equilibrium conditions in a substance from all non-radiating processes, i.e. from various types of collisions of particles in gases and plasma, and from exchanging energies of electron and oscillatory motions in liquids and solid bodies. From the detailed balance of processes follows a spectrum of important physical consequences which are expressed as theorems and laws. They include, first of all, FDT, the Planck law of radiation, the Kirchhoff law of radiation, the Stefan–Boltzmann law of radiation, Boltzmann's distribution of particles over energies, Maxwellian distribution of particles over velocities, the law of energy equidistribution a system's degree of freedom, the ergodic hypothesis. In this case the temperatures, appearing in formulae describing these laws and distributions, are identical in all parts of the equilibrium system and for all sorts of particles, i.e. the temperature *of the whole system* is meant here.

In the real physical reality, however, for the majority of physical bodies the conditions of conservation of thermodynamic equilibrium are absent, generally speaking. This indicates that any physical body emits from its surface some specific portion of the electromagnetic energy which arises inside a body due to physical-chemical reactions, to internal heat sources, to mass transfer inside a body and to other causes. The outgoing energy flux, which exists in such cases, and, accordingly, the gradient (drop) of temperatures between internal and external parts of a system are directly incompatible with the notion of full thermal equilibrium.

The important supposition (hypothesis) on local thermal equilibrium (LTE) is used for similar physical objects. According to this hypothesis, the temperature is different in different elements of a studied medium; there exists the outgoing energy radiation flux (the radiation field is anisotropic), but in this case equilibrium is conserved in very small (elementary) volumes of a medium. But these volumes still contain such a great number of particles (macroscopic particles, molecules, atoms, ions, etc.), that their state can be characterized by the local temperature and other thermodynamic parameters. In their turn, these parameters in macroscopic scales are not constants, but depend on coordinates and time. But in each elementary volume a detailed balance is established, which is determined by the local value of temperature, and in this local scale all physical corollaries of the detailed balance (such as FDT, Boltzmann's and Maxwell's distribution laws, the Kirchhoff law of radiation, etc.) are valid. At the local thermal equilibrium of a medium's elements the state of the medium is one of nonequilibrium, in general. So, under these conditions the thermal radiation is characterized by the value of the temperature at a given point (locally), but the thermal radiation is not in thermal equilibrium with the substance at the scale of the whole body (or medium) under study. In such a case the emission of radiation into the external space and the redistribution of the temperature regime inside a body (or medium) are possible. To maintain the stationary state, in which the gradient thermal field is conserved, the thermal energy losses must be replenished at the expense of extraneous (and, probably, internal) sources.

The reason for the application of the LTE hypothesis to physical objects both on the Earth, and in space lies in the circumstance that the radiation absorbed by an elementary volume of the medium is greatly reprocessed into different forms of energy before it leaves this volume (i.e. is illuminated). As is known from the thermodynamics, such a reprocessing at the scale of the elementary volume proceeds in the direction of establishing thermodynamic equilibrium. So, the whole absorbed portion of radiation energy falling on the opaque solid body is rapidly redistributed over internal energy states in accordance with the local equilibrium distribution inside the solid body. In gases the absorbed radiation energy is redistributed via various kinds of collisions between gas particles: atoms, molecules, electrons and ions. In the majority of cases such a redistribution proceeds fairly rapidly, and the energy levels of gas will be populated in accordance with the equilibrium distribution corresponding to local conditions (Sobolev, 1997).

The local thermal equilibrium is a good approximation to reality for many physical objects and their separate sections. Examples of such objects are: the Earth's atmosphere, surfaces of the Earth, various astrophysical objects. The LTE hypothesis greatly facilitates the calculation of radiation characteristics of such kinds of media (using the so-called LTE models). Certainly, there exists a spectrum of physical conditions in which the LTE assumption is invalid. Examples of such conditions are: (1) highly rarefied gases, in which the rate and efficiency of collisions of particles resulting in redistribution of absorbed energy are low; (2) very rapid nonstationary processes with high gradients of parameters, in the course of which the population density of energy levels has no time to come into correspondence with new conditions; (3) extreme radiation fluxes, in which the absorption of energy and the population density of the upper energy levels are so great, that, owing to collision processes, the equilibrium population density of lower levels will not be achieved. Giving the LTE hypothesis up (in the so-called NLTE models) it becomes necessary to investigate the relations between collision and radiation processes and their influence on the energy distribution between various levels, which represents a fairly complicated problem. Such investigations are carried out in studying shock waves (large gradients), nuclear explosions (non-stationary processes, large gradients, extreme fluxes), gas dynamics of flights at high altitudes and in outer space (very low densities). The greatest deviation from LTE conditions is observed in laser and maser sources, in which the substance with a metastable energy level is excited by the external source. This is because the excited state is metastable and is chosen such that the population density reaches values that essentially differ from equilibrium values (the inverse population density), and is then illuminated into the external space in a coherent manner.

Such problems are of special interest and are not considered in this book. We shall suppose here that the local thermal equilibrium exists in the media we shall investigate later.

Applying the aforementioned radiation laws under local thermal equilibrium conditions to emission and absorption of thermal radiation in physical bodies, we can study radiation transfer processes both inside and outside the physical body, within the framework of the so-called phenomenological theory of radiative transfer (Chapter 9). The significance of this theory for remote sensing problems and astrophysical applications can scarcely be exaggerated. In fact, all fundamental results in remote sensing (and, largely, in the astrophysics) obtained so far are based to an overwhelming extent on the use of the methodology and interpretation of conclusions of the theory of radiation transfer under local thermal equilibrium conditions.