EXISTENCE OF CONCENTRATED WAVES FOR THE DIFFERENT TYPE OF NONLINEARITY.

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Abstract. The nonlinear problem of the wave propagation is considered. In addition to Kerr nonlinearity the question of the existence of concentrated solutions is analyzed for the threshold and saturable nonlinearity. It is shown that both in the case of threshold nonlinearity, and in the case of saturable nonlinearity solitary waves - concentrated solutions of the corresponding wave equations exist.

1. Introduction

Modern radio communications allows so high increasing of radiated signal intensity that it generates modification of medium of propagation and the problem of determining the wave field becomes nonlinear. Theoretical estimates of the effects of ionospheric plasma heating by powerful radiation began to appear long time ago [1]. Experimental confirmation of the interaction of powerful shortwave radiation with ionospheric plasma at oblique propagation [2, 3,] made more active the development of the theory of this interaction [4].

As a rule, the theoretical researches of the nonlinear propagation of powerful wave beams are restricted by models of local nonlinearity. The most commonly used model is the Kerr nonlinearity, in which the nonlinear perturbation of the dielectric permittivity is proportional to the second degree of the modulus of the wave amplitude. This approach allowed us to describe the basic phenomena arising at the nonlinear interaction of radiation with the environment. However such approach has obvious restrictions. Actually, there is no nonlinear effects when the intensity of the wave field is not enough powerful. As soon as the wave amplitude exceeds certain threshold value, so called "disruption" of the environment takes place and the nonlinear dependence of the dielectric permittivity upon the wave field amplitude arises.

Moreover the nonlinear dependence becomes more complex with further growth of the intensity. The upper limit of growth of the dielectric permittivity, namely saturation, is observed for most of the real materials. In this case it is reasonable to use a model medium with saturable nonlinearity.

However, this description of the nonlinear interaction of radiation with the medium is local. This approach is possible only in the case of negligible heat conductivity - when the size of the wave beam is much more than the characteristic scale of the thermal diffusion. Otherwise, you must take into account the spreading of the electron density perturbation from the area of the nonlinear heating. Nonlinear effect becomes nonlocal and, in this case, it is necessary to consider the solution of the diffusion equation for the dielectric permittivity and the solution of the wave equation jointly, as a system.

2. Analysis

In order to describe the wave field in this small area we use the Helmholtz equation for the wave amplitude V.

$$\Delta V + k^2 \cdot \varepsilon \cdot V = 0 ,$$

where k is the wave number and ε is the dielectric permittivity.

For the small wave intensity the dielectric permittivity depends only on the coordinates. Therefore, in order to determine the distribution of the wave field in the space, it is enough to solve the linear problem with the appropriate boundary conditions. But, for the high wave intensity, the dielectric permittivity becomes dependent on the wave amplitude, and it is necessary to solve the nonlinear problem for the description of the wave propagation.

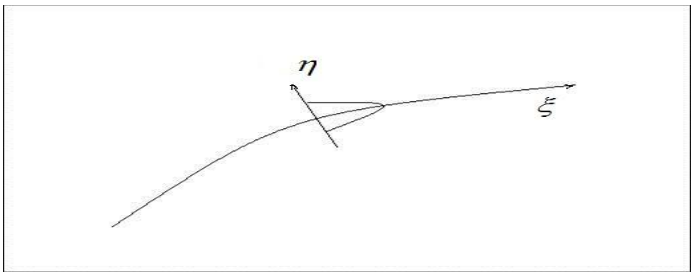

Fig.1. The wave beam and quasi - ray coordinates.

We will consider the propagation of the narrow shortwave beams. Thus we will construct the Helmholtz equation solution concentrated in the small vicinity of the ray trajectory. In this vicinity, we introduce the orthogonal coordinate system: ξ is the length of the trajectory arch; η is the distance along the orthogonal direction to the ray (Fig. 1). We represent the complex-valued function V in the terms of $V = u \cdot \exp(ik\psi)$, where u and ψ - are real functions.

In this approach, the derivatives along the trajectory are essentially less then the derivatives across the ray direction. Therefore, we can write in the main approximation:

$$\frac{\partial^2 u}{\partial \eta^2} + k^2 \left[\varepsilon - \left(\frac{d\psi}{d\xi} \right)^2 \right] u = 0 , \qquad \frac{\partial \psi}{\partial \eta} = 0$$

The electric field heats the environment and creates the perturbation of the dielectric permittivity ε_n. For the slightly inhomogeneous medium we can write the main approximation and we get the typical problem of nonlinear wave propagation.

$\frac{d^2 u}{d\eta^2} = \lambda^2 u - \varepsilon_n \left(u^2 \right) u$, were $\lambda = \frac{d\psi}{d\xi}$ is the phase velocity.

This equation has the first integral.

$$\left(\frac{du}{d\eta} \right)^2 - \lambda^2 u^2 + F(u^2) = E, \quad E - const , \; F(u^2) = \int_0^{u^2} \varepsilon_n(x) dx$$

This equation admits the existence of concentrated solutions if $E = 0$, provided that the equation $F(x) - \lambda^2 x = 0$ has solutions $x = 0$ and $x = x_0 > 0$. Without loss of generality, we choose the center of the solution $\eta = 0$. Then $u = \sqrt{x_0}$, when $\eta = 0$, and $u = 0$ at the infinity. The amplitude maximum of the wave beam is $u_0 = \sqrt{x_0}$, and we can write the formal solution. $y = \pm \int_{u_0}^{u} \frac{dx}{\sqrt{\lambda^2 x^2 - F\left(x^2\right)}}$

2.1 Kerr nonlinearity

In particular, if the correction to the nonlinear dielectric permittivity has the form $\varepsilon_n = \alpha |u|^2$, then $F(u^2) = 0.5 \cdot \alpha \cdot (u^2)^2$, and the equation has a simple root (Fig. 2).

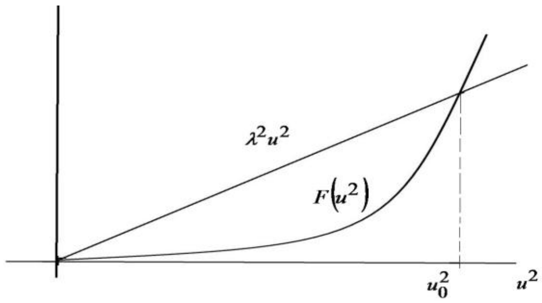

Fig.2. The Ker nonlinearity.

Thus, solution has the form of the simple soliton with the phase velocity $\lambda = \sqrt{0.5 \cdot \alpha \cdot u_0^2}$.

It is a well-known solution. This approach allowed us to describe basic phenomena arising at the nonlinear interaction of radiation with the environment.

2.2 Threshold nonlinearity

However as mentioned above, such approach has obvious restrictions. Actually, there is no nonlinear effects when the intensity of the wave field is not enough powerful. As soon as the wave amplitude exceeds certain threshold value, there is a "disruption" of the environment and the nonlinear dependence of the dielectric permittivity upon the wave field amplitude arises. In this case, we can write the model of the threshold nonlinearity. The nonlinear perturbation of the dielectric permittivity for this situation can be represent by the formula. $\varepsilon_n(|u|^2) = \alpha |u|^2 \cdot \theta\left(|u|^2 - A^2\right)$, $\theta(x)$ - is the Heaviside theta function, A is the threshold value.

In this case $F(|u|^2) = \int_0^{|u|^2} \varepsilon_n(t)dt = \dfrac{\alpha}{2}\theta\left(|u|^2 - A^2\right) * (|u|^4 - A^4)$, and the expression (1) results in the elliptic integral. Nevertheless it is obvious (Fig. 3) that in this case, we have also the concentrated solution with the phase velocity $\lambda = \sqrt{0.5 \cdot \alpha \cdot \left(u_0^2 - A^4 / u_0^2\right)}$.

The concentrated solution with the threshold nonlinearity is very close to the usual soliton, but it is somewhat narrower in the center of the beam and it has a "long" tails. The interaction between these beams differs from the interaction of the soliton collisions.

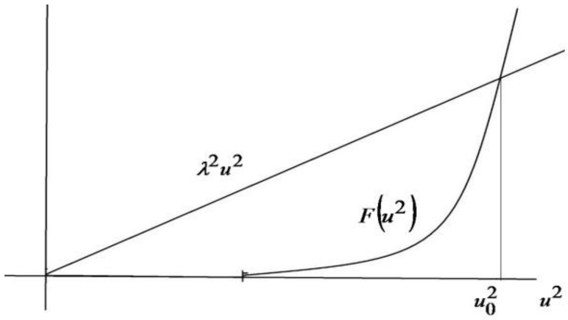

Fig.3. The threshold nonlinearity

2.3 Saturable nonlinearity

The nonlinear dependence becomes more complex with magnification of the wave power. The upper limit, of the dielectric permittivity is observed for most of real materials. In this case it is reasonable to use a model medium with saturable nonlinearity, in which the dependence of the dielectric permittivity on the wave intensity is described by fractional-linear function [5, 6].

The perturbation of the dielectric permittivity $\varepsilon_n = \dfrac{\alpha|u|^2}{1+\beta|u|^2}$.

In this case $F(|u|^2) = \int_0^{|u|^2} \varepsilon_N(t)dt = \dfrac{\alpha|u|^2}{\beta} - \dfrac{\alpha}{\beta^2}\ln(1+\beta|u|^2)$

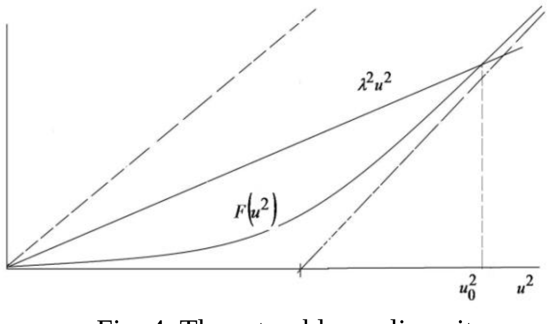

Fig. 4. The saturable nonlinearity

We can see from Fig. 4 that it is also possible the existence of the concentrated solutions. The phase velocity has the limit $\lambda = \sqrt{\alpha/\beta}$, when the wave intensity tends to the infinity. In this case, the environment becomes linear. However, at any finite amplitude, the concentrated solution exists.

2.4 Nonlocal nonlinearity

The previous descriptions of the nonlinear problems were local. That approaches are possible only in the case of negligible heat conductivity, when the size of the wave beam is much more than the characteristic scale of the thermal diffusion. Otherwise, it is necessary to take into account the spreading of the electron density perturbation from the area of the nonlinear heating. Nonlinear effect becomes nonlocal [10]. In this case for the ε_n, we have to write the diffusion equation.

The computational solution allow us to assert that the concentrated solution exist for any value of the diffusion scale. The concentrated solution has not any singularities for any value of the scale of the diffusion process and it looks like the soliton but more wide. Naturally, the diffusion process enlarges the soliton.

5. References

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