ELEMENTARY CONVECTIVE CELL IN INCOMPRESSIBLE VISCOUS FLUID AND ITS PHYSICAL PROPERTIES

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Abstract. The energy principle of convective structures formation in a layer of viscous incompressible fluid uniformly heated from below is proposed. It is offered to use an elementary cylindrical convective cell which parameters are determined in the following article. The results of experiments on the determination of the oil movement in the elementary cell convective velocity are presented.

1. Introduction

First mathematical description of equilibrium of the horizontal layer of fluid heated from below with the free boundaries was proposed by Lord Rayleigh in 1916 [1,2]. Critical numbers: the Rayleigh number R_{min} = 657.51 and the wavenumber k_{min} = 2.221 were found as a solution of this problem for the main perturbation ($n = 1$) and the Prandtl number $\Pr \approx 1$ in the spermaceti thin layer.

In [2-4] it is shown that solutions describing the horizontal and vertical velocity components inside the cell, as a result of certain geometric transformations may form convective horizontal shafts, square convective cells or regular polygons. These structures tend to completely fill the surface of the convective layer and ensure maximum heat transfer between the boundaries of the layer.

From our point of view, the principle of polygonal convective structures formation shouldn’t be the geometric, but an energy one [5], which states that the higher temperature of the tank bottom with the appropriate temperature gradient leads to the convective cells amount increasing with the shapes close to a polygonal (especially hexagonal) making heat exchange between lower and top boundaries more efficient. To implement this principle of polygonal convection cells generation it is necessary to introduce the concept of an elementary convective cell which under dense packing form the polygonal convective structure. The vertical velocity component of the elementary convective cell is described by the cylindrical Bessel function of the first kind of zero order [5] and the physical properties of the proposed cell correspond to the experimental data obtained in small containers [6].

2. Theory of the elementary convective cell with free boundaries [5]

To describe the convective processes in a horizontal layer of a viscous, incompressible fluid heated from below let's take the initial system as the Navier-Stokes equations in the Boussinesq approximation, recorded for the dimensionless perturbation $\vec{\mathrm{v}}, T$ and dimensionless variables:

$$\frac{\partial}{\partial t}\Delta \mathrm{v}_z = \Delta\Delta \mathrm{v}_z + R\Delta_\perp T,\; P\frac{\partial T}{\partial t} = \Delta T + \mathrm{v}_z \qquad (1), (2)$$

where $\Delta = \Delta_\perp + \frac{\partial^2}{\partial z^2}$ - the Laplace operator, $\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$ - transverse Laplacian, z-axis points upward, perpendicular to the layer boundaries $z = 0$ and $z = 1$, $R = g\beta h^3\Theta(\nu\chi)^{-1}$ - the Rayleigh number, g – gravitational acceleration, directed against the axis z, $P = \nu\chi^{-1}$ - the Prandtl number, ν and χ - the kinematic viscosity and thermal conductivity of fluid, β - the coefficient of volume thermal expansion of fluid, $\vec{\mathrm{v}} \equiv (\mathrm{v}_r, \mathrm{v}_z)$, T - the perturbation of velocity, pressure and temperature respectively.

To determine the "normal" perturbations the equations (1) (2) must be supplemented by the boundary conditions. We consider free cell boundaries, i.e. shear stresses and temperature perturbations at the boundaries both equal zero: $\partial \mathrm{v}_r / \partial z = 0, T = 0$. These requirements are provided by the following equations at the layer boundaries $z = 0$ and $z = 1$:

$$\mathrm{v}_z = 0, \frac{\partial^2 \mathrm{v}_z}{\partial z^2} = 0, T = 0 \qquad (3)$$

Equations (1) and (2) have partial solutions, which describe the temporal dynamics of the vertical velocity's and temperature's perturbations in the axially symmetric cylindrical cell:

$$\mathrm{v}_z\left(r,z,t\right)=\mathrm{v}\left(z\right)J_0\left(k_r r\right),\,T\left(r,z,t\right)=\vartheta\left(z\right)J_0\left(k_r r\right) \qquad (4), (5)$$

where at the right sides the multiplier $\exp\left(-\lambda t\right)$ is omitted, λ - the eigenvalues characterizing the change in time perturbations (4), (5); $\mathrm{v}\left(z\right)$ and $\vartheta\left(z\right)$ - perturbations' amplitudes of the vertical velocity and temperature respectively; $J_0\left(x\right)$ - the Bessel functions of the first kind of zero-order of the argument x; k_r - the radial wave number characterizing perturbations dependence on the transverse coordinate r.

From the boundary conditions (3) one can obtain the conditions imposed on the vertical velocity and temperature perturbations' amplitude:

$$\mathrm{v}\left(0\right)=\mathrm{v}\left(1\right)=0,\ \frac{\partial^2\mathrm{v}\left(0\right)}{\partial z^2}=\frac{\partial^2\mathrm{v}\left(1\right)}{\partial z^2}=0,\ \vartheta\left(0\right)=\vartheta\left(1\right)=0 \qquad (6)$$

From (6), as shown in [3], the vertical velocity and temperature perturbations' amplitudes can be represented as a simple harmonics:

$$\mathrm{v}\left(z\right)=A\cdot sin\left(n\pi z\right),\,\vartheta\left(z\right)=B\cdot sin\left(n\pi z\right) \qquad (7)$$

where $n=1,2,3,...$ - integers, A and B - constant coefficients.

On the strength of (4), (7) and the fluid incompressibility condition it follows that the expressions for the vertical $\mathrm{v}_z\left(r,z,t\right)$ and radial $\mathrm{v}_r\left(r,z,t\right)$ velocities of convective flow in the cell, multiplying by $\mathrm{e}^{-\lambda t}$, can be represented as such expressions:

$$\mathrm{v}_z=A\cdot sin\left(n\pi z\right)J_0\left(k_r r\right),\mathrm{v}_r=-An\pi k_r^{-1}\cos\left(n\pi z\right)J_1\left(k_r r\right). \qquad (8)-(9)$$

The solution (9) corresponds to the physically reasonable boundary conditions on the axis of the cell ($r=0$) and on its outer boundary ($r=R_c$) stating that the radial velocity of the fluid must be equal to zero. From this we can determine the value of the radial wave number:

$$k_r = \sigma_{1,i} R_c^{-1} \qquad (10)$$

where R_c - convective cell radius divided by the depth of the layer, $\sigma_{1,i}$ - i - th zero of the Bessel function of the first order, $i = 1, 2, 3, ...$. In particular, the first five zeros of Bessel functions are defined as follows [7]: $\sigma_{1,1} = 3.832$; $\sigma_{1,2} = 7.016$; $\sigma_{1,3} = 10.173$; $\sigma_{1,4} = 13.324$; $\sigma_{1,5} = 16.471$.

Note that the solution (4) is also obtained in [6,8,9]. E.g., in [8] it is shown that the radial wavenumber is determined as the ratio of the critical wavenumber $a = 2.682$ of the asymmetric boundary conditions problem (the notation of the cited work is kept) to the depth of the layer: $k_r = a/h$.

3. Experimental data and the velocity of convective heat transfer in a cell measurement

During examination of the Benard cells the question of the magnitude of the horizontal velocity of convective flow on the cell surface is important. To determine its value a series of experiments have been carried out in which used vacuum oil BM - 5 (2 ml of the weight 1.8 g) and a small amount of aluminum powder (0.056 g) were utilized to create Benard cells. The oil was heated from below by an electric furnace, the temperature was maintained at about 130 ± 1°C at the bottom of the container. The probe made of two parallel cylindrical thin wires of 0.04 mm diameter copper was lowered vertically in one of the cells on the top half of the radius. The length of one probe was 4.3 mm and the other was 5.4 mm. These probes were attached to a metal rod, which was located a protractor to determine the angle of deflection of the longer probe when they were shortly immersed in the oil.

As seen from the experimental data, after dipping the probe into the cell at the half of the radius the deflection angle about 1±0,1º is formed. Long probe’s calibration was carried out using a vertical jet of water flowing freely from a tap. Adjusting the water flow from the tap and measuring the diameter of the water jet, mass of the water and water’s flowing time it is possible to calculate the flow velocity:

$$V_V = 4M / (\pi d^2 \rho t)^{-1} \qquad (11)$$

where M - mass trapped in the water tank, d - diameter of the water jet, ρ - density of water, t - time of water leakage into the tank.

The angle of deflection of a long probe immersed into the water jet perpendicular to the cylindrical surface corresponds to the flow velocity calculated on the formula (11). To clarify the analytic dependence of the probe's deflection angle from the water flow rate, let's assume that the probe has a cylindrical shape. In this case, the resistance force of the probe to the oncoming flow of water per unit length is defined by Stokes - Oseen formula [10]:

$$F = 4\pi\mu_V V_V \cdot (ln(7,406 \cdot R_e^{-1}))^{-1} \quad (12)$$

where μ_V - the dynamic viscosity of water, $R_e = \rho V_V d_{pr} \mu_V^{-1}$ - Reynolds number.

Using the experimental data for the water and taking into account the fact that the water velocity into the oil velocity transition factor is determined by the rate $V_V / V_{Oil} = \mu_{Oil}\kappa_V / \left(\mu_V \kappa_{Oil}\right) \approx 10.0$ the convective oil velocity dependence from the probe's angle of deflection was plotted.

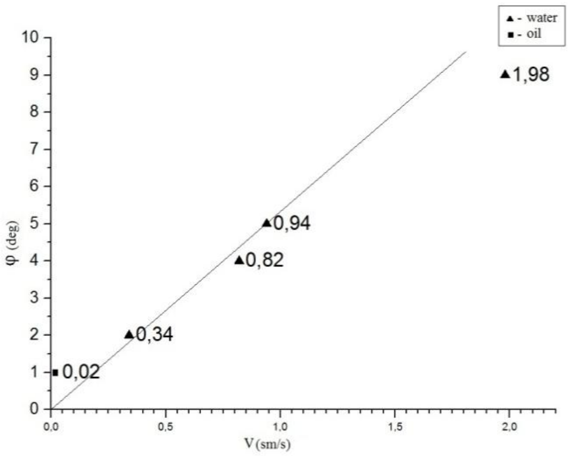

Fig. 1. Convective water velocity dependence from the probe's angle of deflection.

The Fig. 1 shows that the velocity of the oil flow on the surface of the convective cell at half radius from the axis is about $V_{Oil} \approx 0.02$ cm / sec.

Another method for changing the mass transfer rate is based on a visual determination of the velocity chosen particles (look at the Fig. 2) from the axis position in the cell. The measurements were performed using a time - lapse video scan. The Fig. 3 shows an example of measuring the velocity of particles chosen from the position in the cell. Estimates show that this speed determined by the value about $V_{Oil} \approx 0.43$ cm/sec.

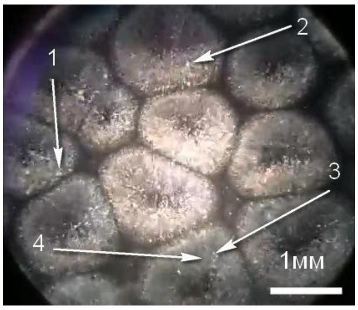

Fig. 2. Type of convective cells and location of markers.

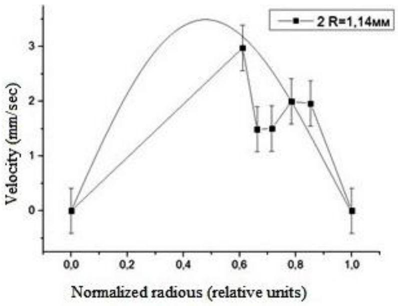

Fig. 3. Horizontal velocity particle with a radius of cell R_c = 1,14 mm.

Conclusions

An energy principle of convective cells formation in a layer of viscous incompressible fluid with uniform heating from below is proposed in this paper. The parameters of the convection cell are determined. The velocity of oil flow on the surface of the convective cells has been measured and consist value for different sizes from $V_{Oil} \approx 0.02$ cm / sec to $V_{Oil} \approx 0.43$ cm / sec.

Sources

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