



# Influence of plasma on gravitational lensing effects

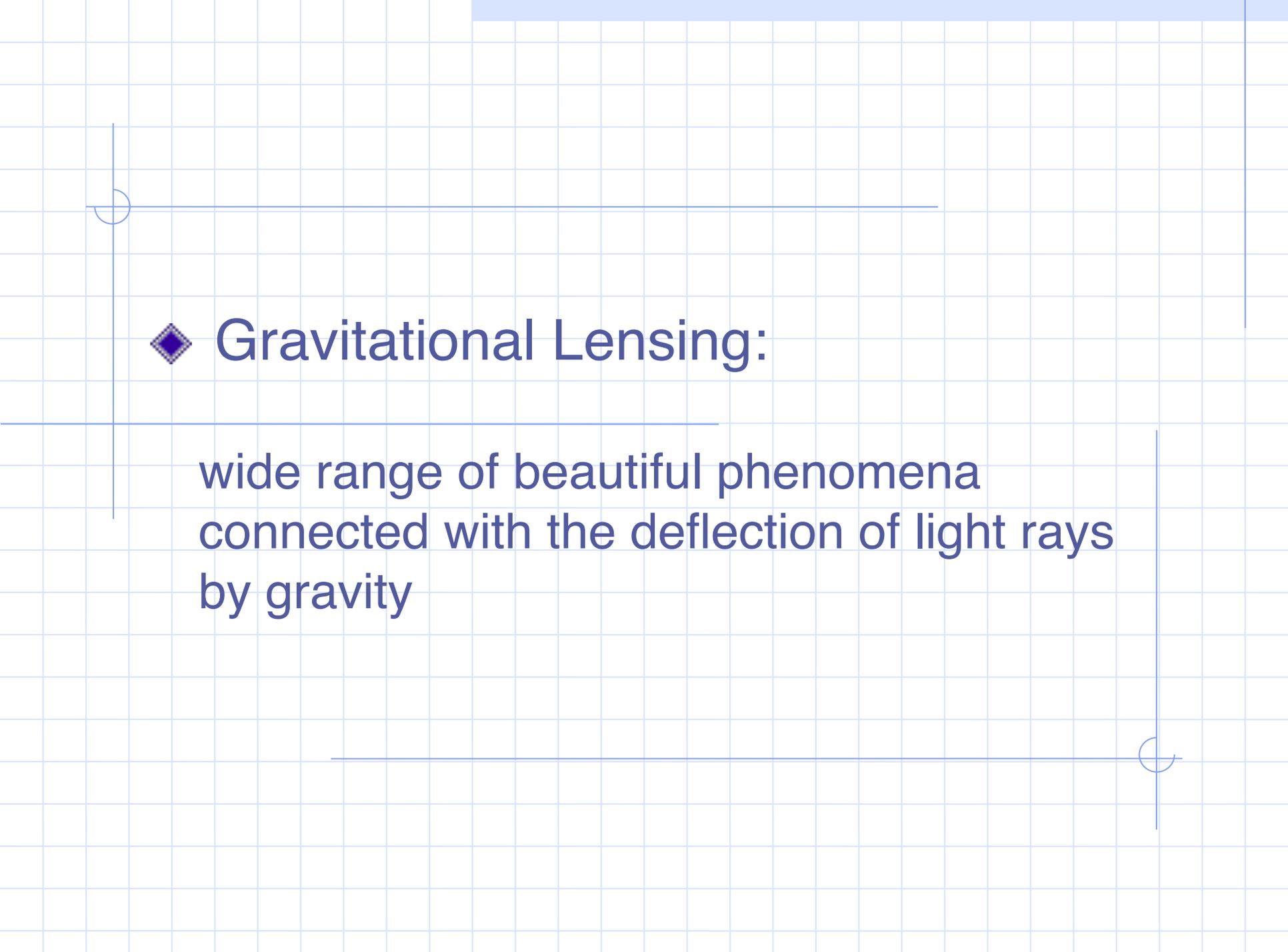
O.Yu. Tsupko<sup>1,2</sup>,  
G.S. Bisnovatyi-Kogan<sup>1,2</sup> and V. Perlick<sup>3</sup>

<sup>1</sup>Space Research Institute (IKI) of Russian Academy of Sciences,  
Moscow, Russia

<sup>2</sup>National Research Nuclear University MEPhI, Moscow, Russia

<sup>3</sup>ZARM, University of Bremen, Germany

e-mails: [tsupko@iki.rssi.ru](mailto:tsupko@iki.rssi.ru), [gkogan@iki.rssi.ru](mailto:gkogan@iki.rssi.ru), [perlick@zarm.uni-bremen.de](mailto:perlick@zarm.uni-bremen.de)



## ◆ Gravitational Lensing:

wide range of beautiful phenomena  
connected with the deflection of light rays  
by gravity

- ◆ The Castle on the Mall in Washington, D.C., as viewed from the Natural History museum.



- ◆ Photo credit: Smithsonian Photo by Eric Long.
- ◆ <https://www.cfa.harvard.edu/~bmcleod/castle.html>

- ◆ Now we place a black hole with the mass of Saturn over the middle of the Mall, and view the Castle through the resulting gravitational lens.

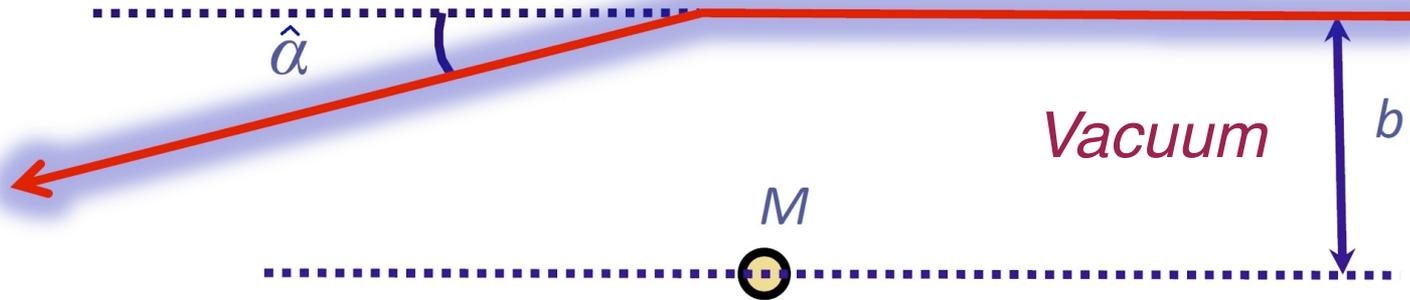


- ◆ Note that there are two images of each of the middle towers, one inside the ring and one outside. The inner image is turned inside-out. <https://www.cfa.harvard.edu/~bmcleod/castle.html>

# Vacuum:

Einstein angle:

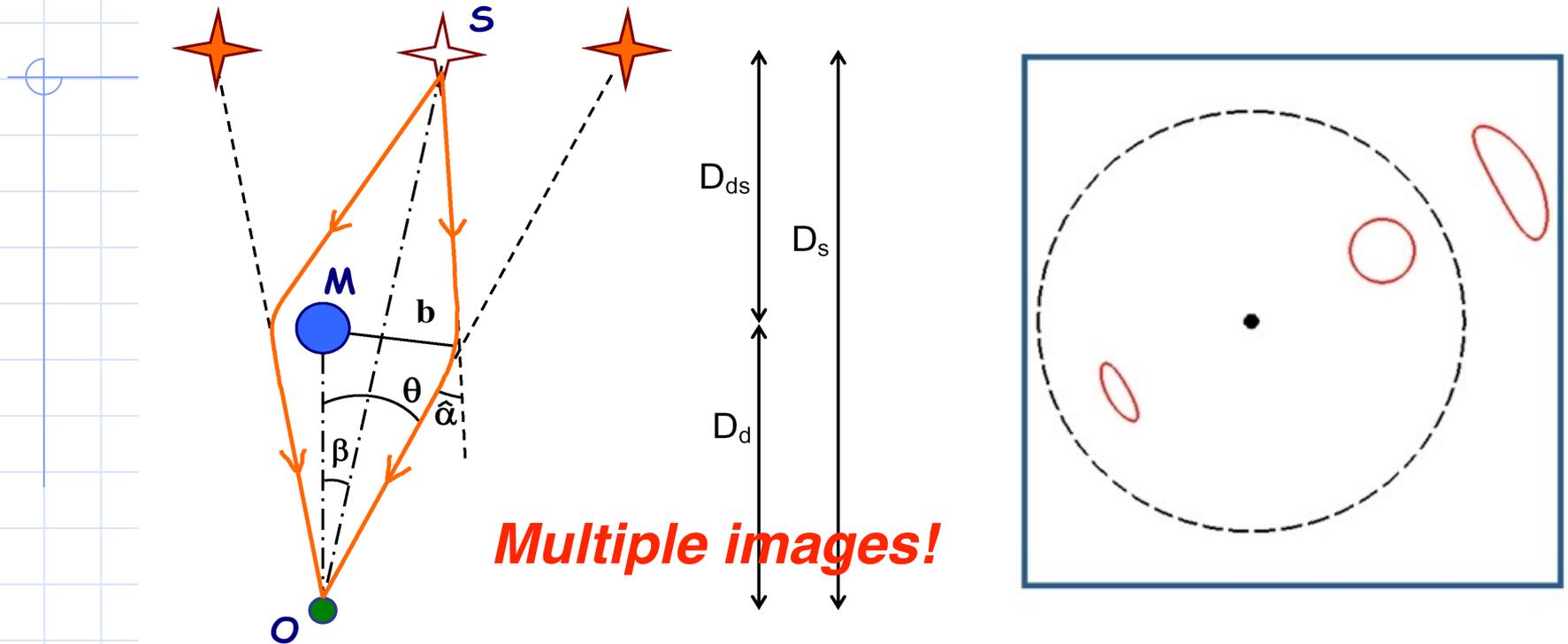
$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{2R_S}{b} \quad b \gg R_S = \frac{2GM}{c^2}$$



Deflection angle of the photon in vacuum does not depend on the photon frequency (or energy). Deflection in vacuum is **achromatic**.

**Gravitational lensing in vacuum is achromatic.**

# Gravitational lensing in vacuum, the simplest scheme



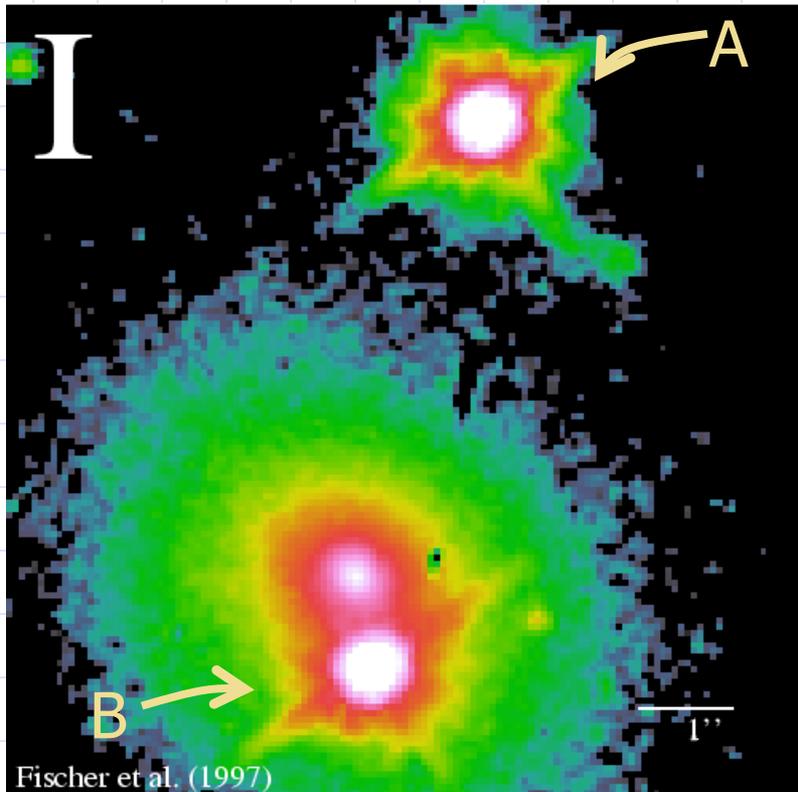
At this picture there is the example of the simplest model of **Schwarzschild point-mass lens**.

Observer sees **two point images** of source **instead of one** single real point source.

Rays from source which are not in plane of Source-Lens-Observer will not go to observer.

*Note: point-mass lens is never used in modelling of observed system with several images. It is just the simplest model, useful for explanation of many effects*

# The first observational example of gravitational lens (1979)

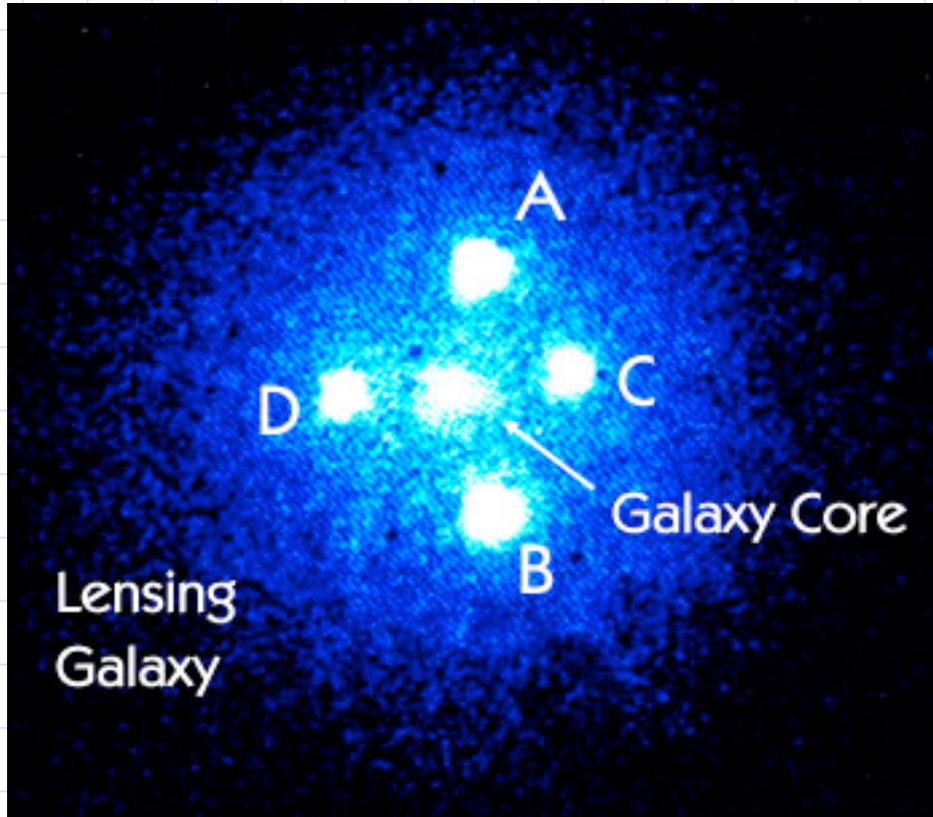


- ◆ Maximum separation – 6.1 arcsec
- ◆ Image redshift – 1.41
- ◆ Lens redshift – 0.36
- ◆  $B/A = 2/3$

QSO 0957+561

Note: lens here is not point-mass but has more complicated structure

# The Einstein Cross



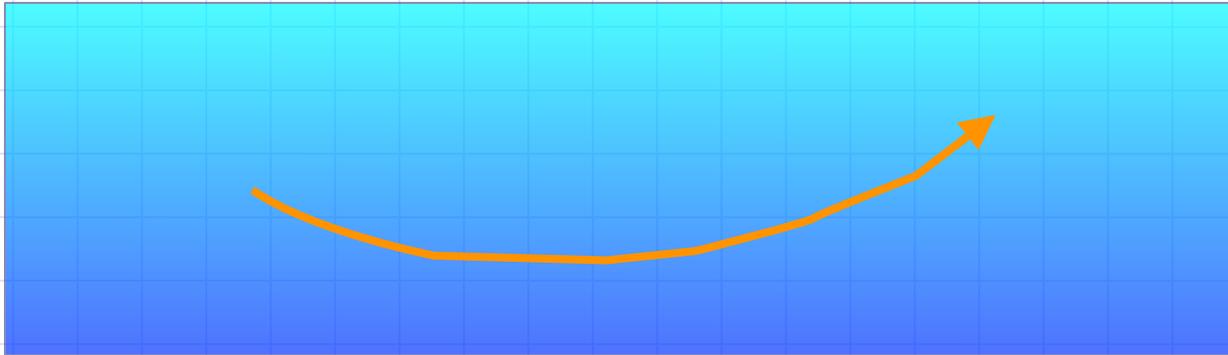
- ◆ It is observed: four QSO images arrayed around the nucleus of the galaxy.

Now more than hundred of such system (so called strong lens system), with several images of the same distant source

# Plasma:

How is this situation changed in presence of *plasma*?  
In plasma two effects should be taken into account:

## 1) Effect of refractive deflection in non-homogeneous medium



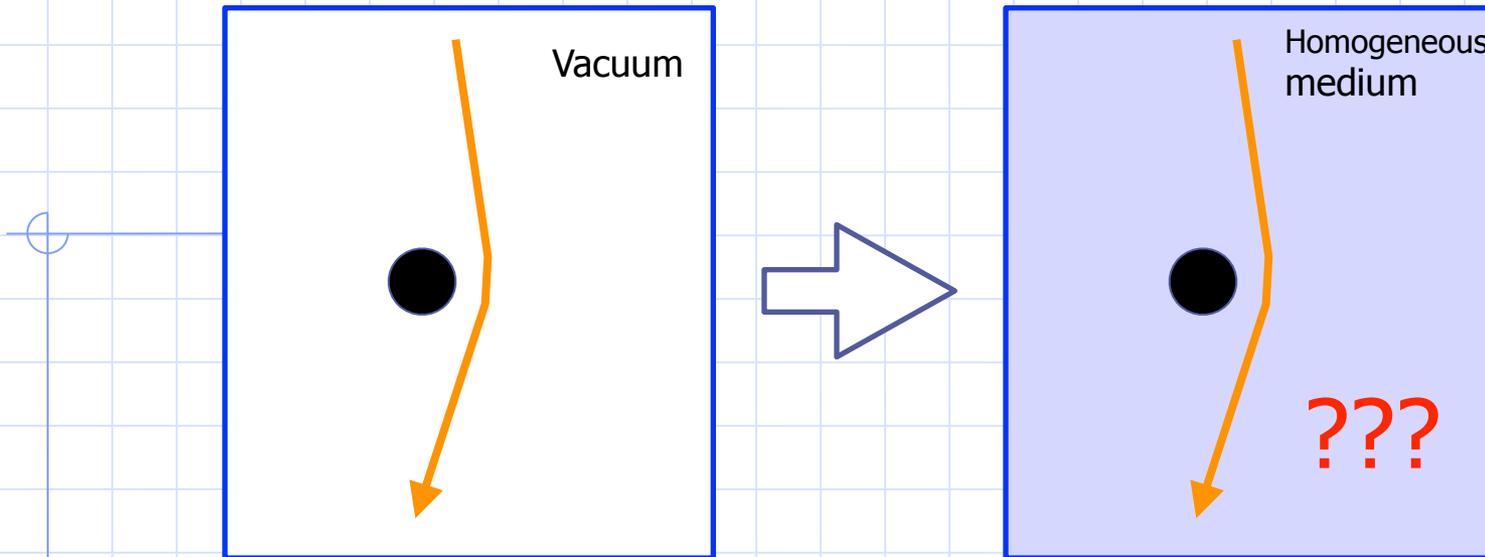
$$n=n(x)$$

'Non-homogeneous' means here that the refractive index depends explicitly on space coordinates

## 2) What about the gravitational deflection itself?

Let us consider a homogeneous medium, so there is no refractive deflection.

Is the gravitational deflection of the light rays in the medium the same as in vacuum?



is the gravitational deflection of the light rays in the medium the same as in vacuum?

We mean here the *homogeneous* medium, so there is no refraction.

Answer is:

deflection is **the same** as in vacuum, if medium is **non-dispersive** ( $n=\text{const}$ )

And it **differs**, if medium is **dispersive!** ( $n=n(\omega)$ )

The physical reason is a dependence of the wave frequency on space coordinates in presence of gravity (gravitational redshift)

# What about plasma?

## Gravitational deflection of light rays in presence of uniform plasma

Refraction index  
of plasma:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(r)]^2}, \quad \omega_e^2 = \frac{4\pi e^2 N_0}{m} = \text{const}$$

Here, the frequency of the photon  $\omega(r)$  depends on the spatial coordinate  $r$  due to the presence of a gravitational field (the gravitational redshift). We will use the following notation:  $\omega(\infty) \equiv \omega$ ,  $e$  is the charge of the electron,  $m$  is the electron mass,  $\omega_e$  is the electron plasma frequency, and  $N_0 = \text{const}$  is the electron concentration in a homogeneous plasma. This for-

**We have shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:**

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

in vacuum



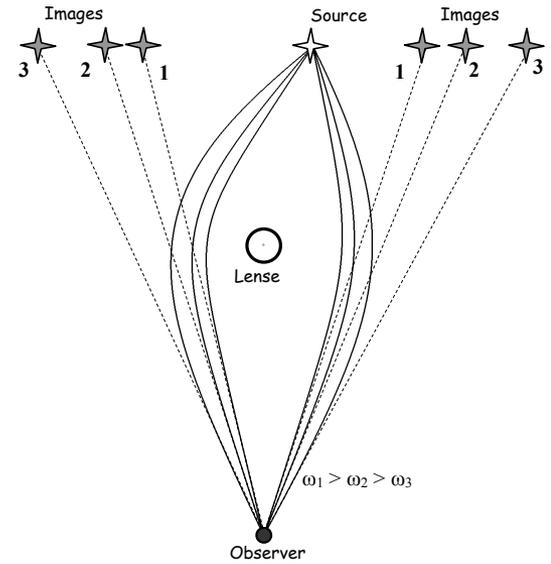
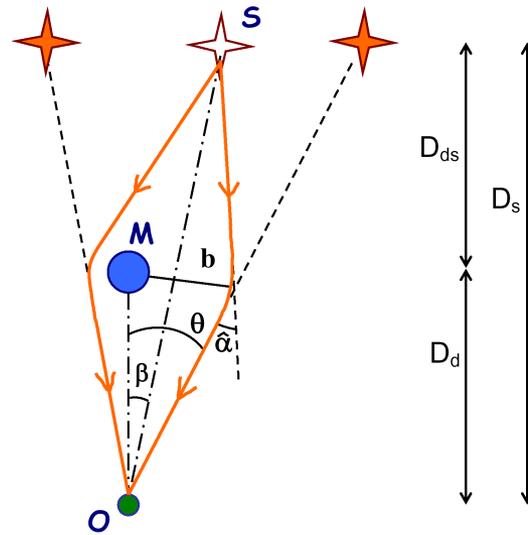
$$\hat{\alpha} = \frac{R_S}{b} \left( 1 + \frac{1}{1 - (\omega_e^2/\omega^2)} \right)$$

in homogeneous plasma

***Chromatic gravitational deflection!***

# Effect of 'Gravitational radiospectrometer'

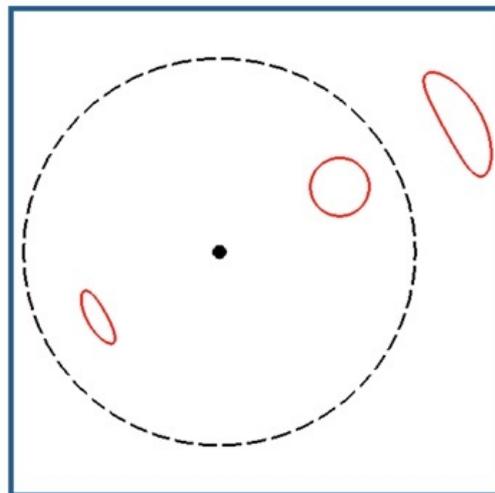
Instead of two concentrated images with complicated spectra, we will have two 'rainbow' images, formed by the photons with different frequencies, which are deflected by different angles.



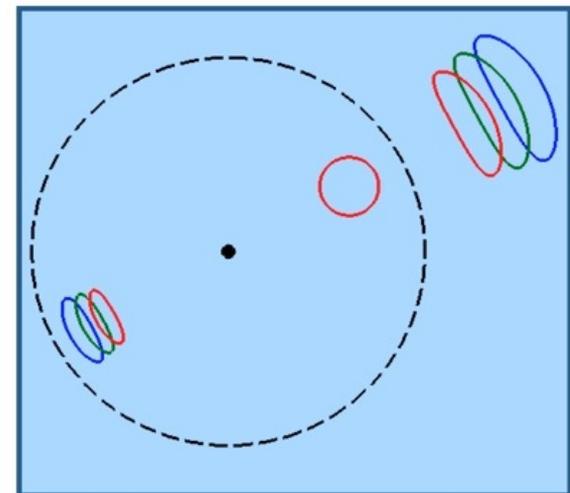
Point-mass gravitational lens in homogeneous plasma: it acts like spectrometer!

Effect is significant only for radiowaves

vacuum



homogeneous plasma



Different colors mean different wavelengths

## Gravitational lensing in non-homogeneous plasma:

In non-homogeneous plasma two effects should be taken into account:

- 1) Difference of gravitational deflection from vacuum case due to plasma presence
- 2) Refraction (usually bigger)

Both effects are chromatic in plasma

# Total deflection angle (in weak deflection approximation)

gravitational deflection in plasma

$$\hat{\alpha} = \alpha_{einst} + \alpha_{add} + \alpha_{refr}$$

$= \frac{2R_S}{b} \quad \propto \frac{R_S}{b} \frac{\omega_e^2}{\omega^2} \quad \propto \nabla \frac{\omega_e^2}{\omega^2}$

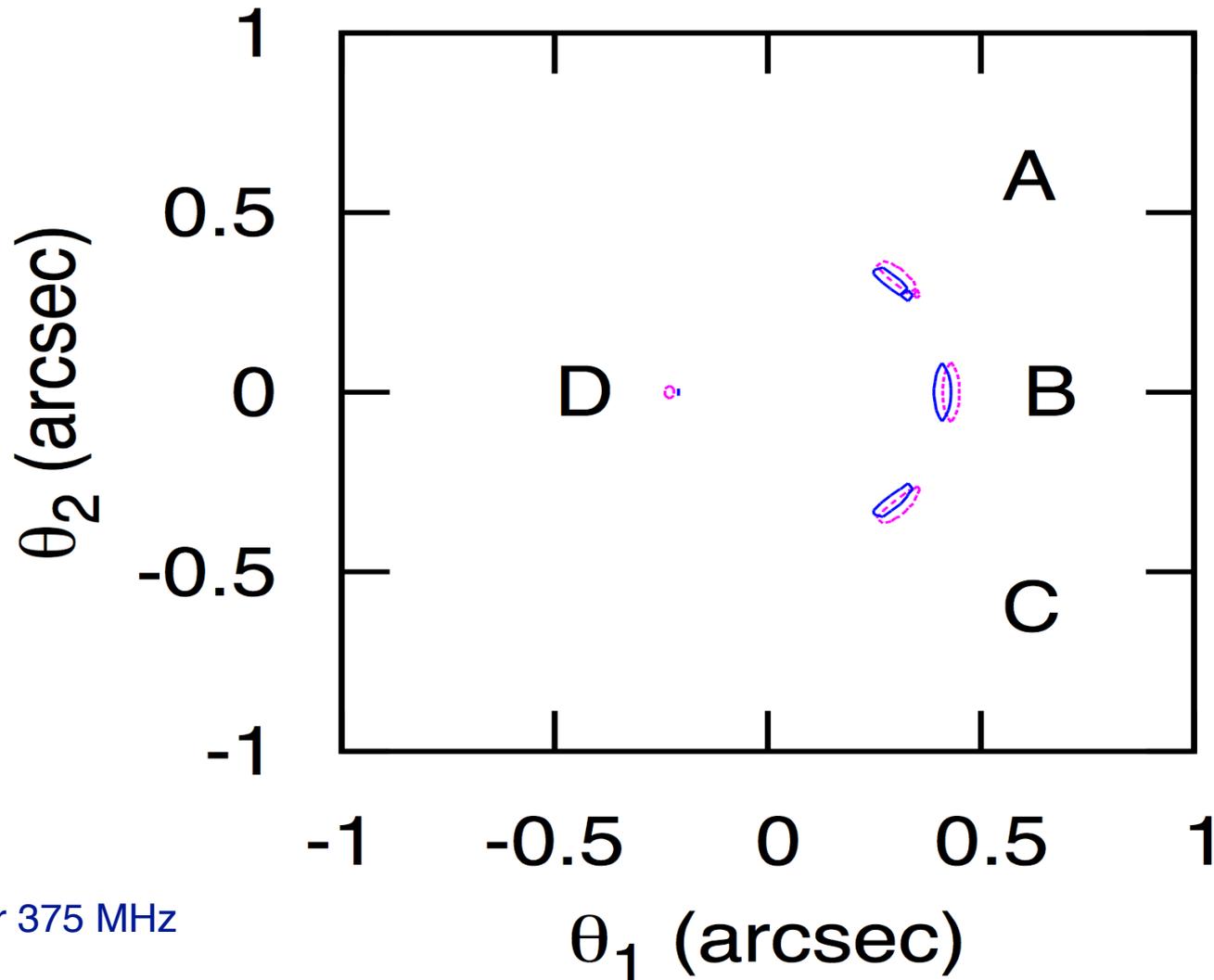
*Vacuum gravitational deflection (Einstein)*

*Additional correction to the gravitational deflection due to plasma presence. It depends on the photon frequency. It takes place both in homogeneous and inhomogeneous plasma*

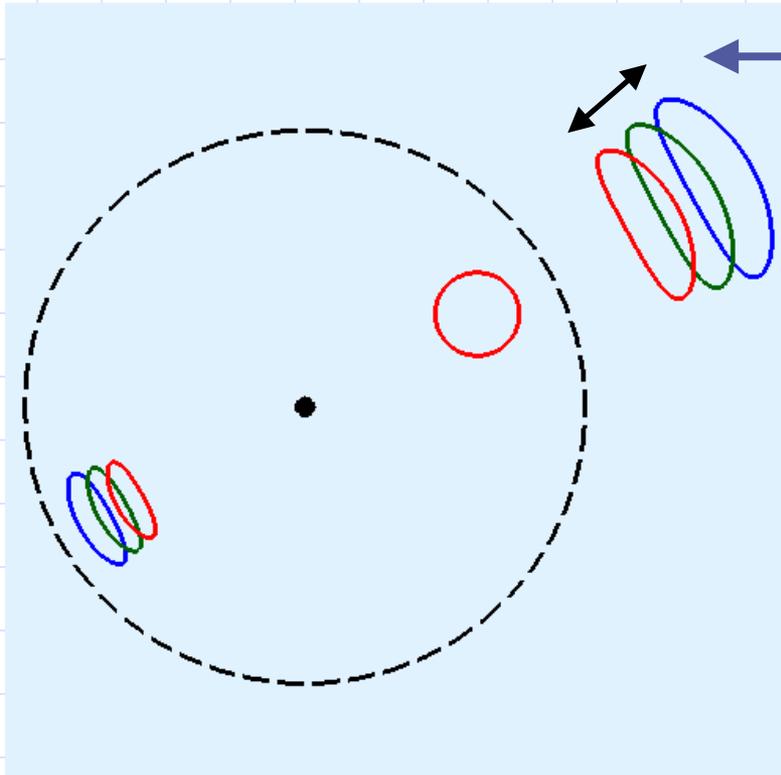
*The refraction connected with the plasma inhomogeneity. It depends on the photon frequency because the plasma is dispersive medium. This angle equals to zero if the plasma is homogeneous.*

# Observational predictions:

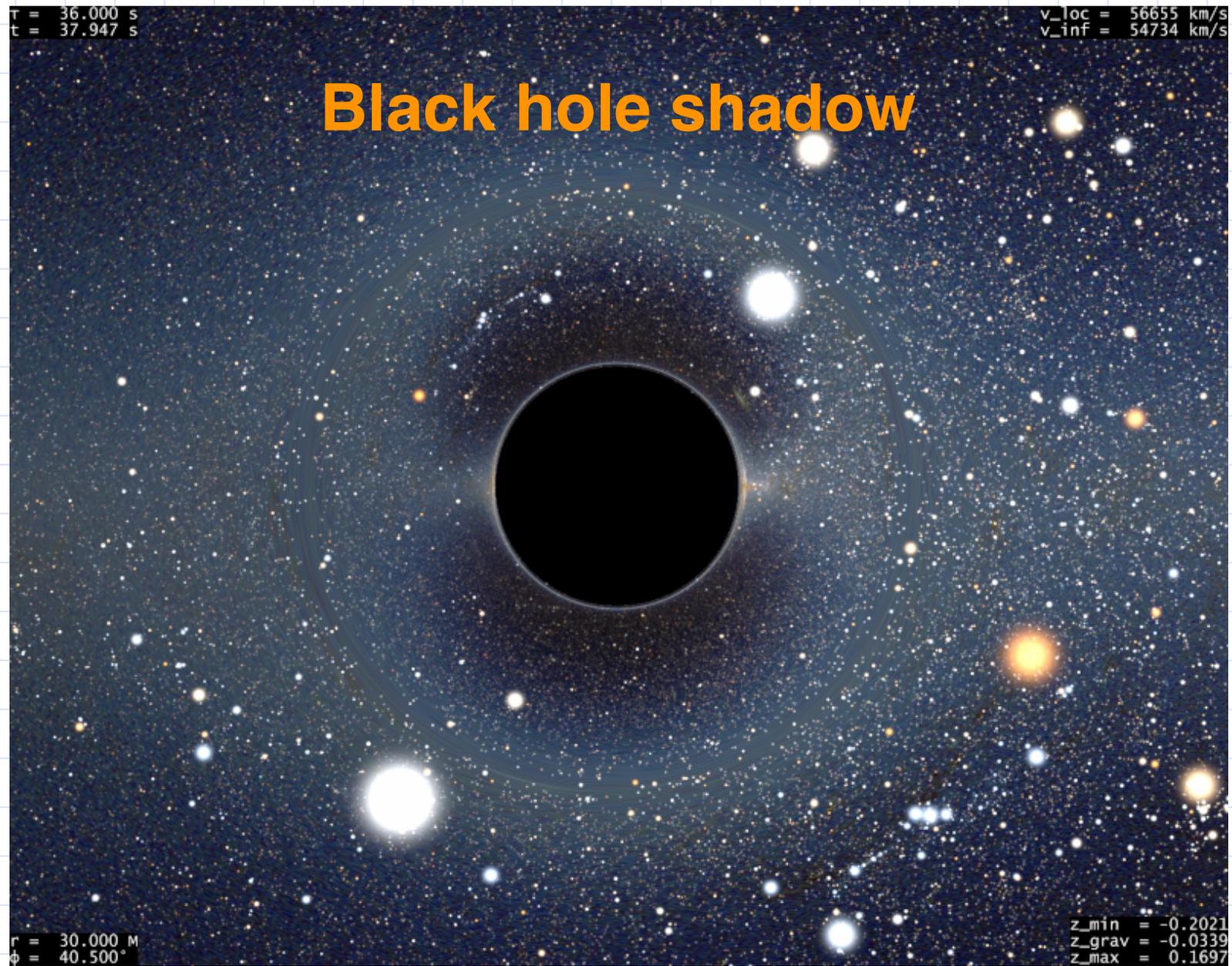
- ◆ Xinzhong Er, Shude Mao, 2014: angular difference between optical and radio images is up to  $10^{-2}$  arcsec due to presence of inhomogeneous plasma



# What we propose for observations:



- 1) Compare observations of strong lens system with multiple images in optical and radio band, or compare observations in two radio bands
- 2) Shift of angular position of every image can be observed
- 3) As a result: investigation of plasma properties in vicinity of lens



Riazuelo A 2014 Simulation of starlight lensed by a camera orbiting a Schwarzschild black hole ([www2.iap.fr/users/riazuelo/interstellar](http://www2.iap.fr/users/riazuelo/interstellar))

# Shadow of BH, influence of plasma on its size

V. Perlick, O.Yu. Tsupko, G.S. Bisnovatyi-Kogan, Phys. Rev. D 92, 104031 (2015)

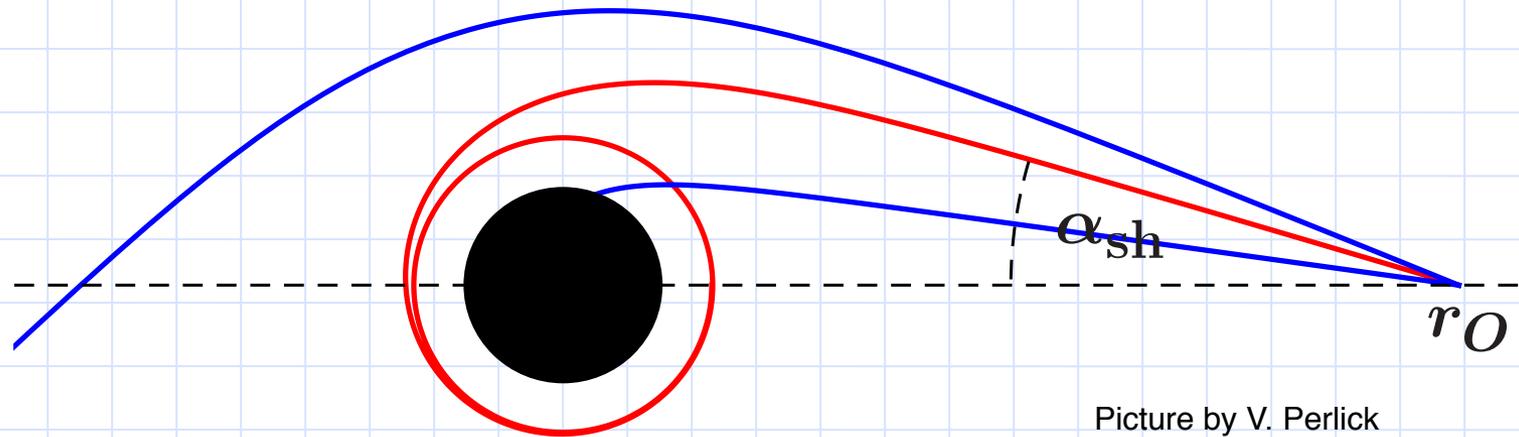
SMBH in the center of galaxies, for example, in the center of our galaxy

A distant observer should “see” this black hole as a dark disk in the sky which is known as the “shadow”

For the black hole at the center of our galaxy, size of the shadow is about  $53 \mu\text{as}$  (size of grapefruit on the Moon).

At present, two projects are under way to observe this shadow which would give important information on the compact object at the center of our galaxy. These projects, which are going to use (sub)millimeter VLBI observations with radio telescopes distributed over the Earth, are the Event Horizon Telescope (<http://eventhorizontelescope.org>) and the BlackHoleCam (<http://blackholecam.org>).

On the theoretical side, the shadow is defined as the region of the observer's sky that is left dark if there are light sources distributed everywhere but not between the observer and the black hole.



Note: The shadow is *not an image of the event horizon*. The boundary of the shadow corresponds to light rays that asymptotically approach the photon sphere (at  $r = 3M$  in the Schwarzschild case) and not the horizon (at  $r = 2M$  in the Schwarzschild case). Moreover, light rays are bent due to gravity. So shadow is the image of region inside the photon sphere increased by light bending.

We would like to investigate the influence of matter around of BH on the observed size of the shadow. Modeling is made by different groups.

We perform the first attempt of analytical investigation of plasma influence on the shadow size, in frame of geometrical optics, taking into account effects of general relativity and plasma presence.

We have derived compact analytical formula for angular size of the shadow of BH surrounded by spherically symmetric plasma distribution

**Angular radius  $\alpha_{\text{sh}}$  of shadow:**

$$\sin^2 \alpha_{\text{sh}} = \frac{h(r_{\text{ph}})^2}{h(r_{\text{O}})^2}$$

**Condition for photon sphere:**

$$\left. \frac{d}{dr} h(r)^2 \right|_{r=r_{\text{ph}}} = 0 \quad h(r)^2 = r^2 \left( \frac{r}{r - 2M} - \frac{\omega_p(r)^2}{\omega_0^2} \right)$$

Main results for the shadow:

In the presence of a plasma the size of the shadow **depends on the wavelength** at which the observation is made, in contrast to the vacuum case where it is the same for all wavelengths.

The effect of the plasma is significant only in the **radio** regime.

For an observer far away from a Schwarzschild black hole the plasma has a **decreasing effect** on the size of the shadow.

## Conclusions:

1. In presence of both gravity and plasma the deflection angle is physically defined by mutual combination of different phenomena: gravity, dispersion, refraction.

2. In weak deflection approximation two effects should be taken into account:  
- difference of gravitational deflection from vacuum case  
- refractive deflection (usually bigger)

*Presence of plasma always makes gravitational lensing chromatic*

3. It leads to difference in angular position of the same image at different (radio) wavelengths, up to milliarcseconds

4. Shadow of BH becomes chromatic and smaller due to plasma.

## Publications:

1. G.S. Bisnovaty-Kogan, O.Yu. Tsupko, *Gravitation and Cosmology*, 15(1), 20-27 (2009).

2. G.S. Bisnovaty-Kogan and O.Yu. Tsupko, *MNRAS* 404, 1790–1800 (2010)

3. O.Yu. Tsupko and G.S. Bisnovaty-Kogan, *Physical Review D* 87, 124009 (2013)

4. O. Yu. Tsupko, *Physical Review D* 89, 084075 (2014)

5. G. S. Bisnovaty-Kogan and O. Yu. Tsupko, *Plasma Physics Reports*, 2015, Vol. 41, No. 7, pp. 562–581 (review, arXiv:1507.08545)

6. V. Perlick, O.Yu. Tsupko and G.S. Bisnovaty-Kogan, *Physical Review D* 92, 104031 (2015)