



# Magnetic acceleration of ultrarelativistic jets

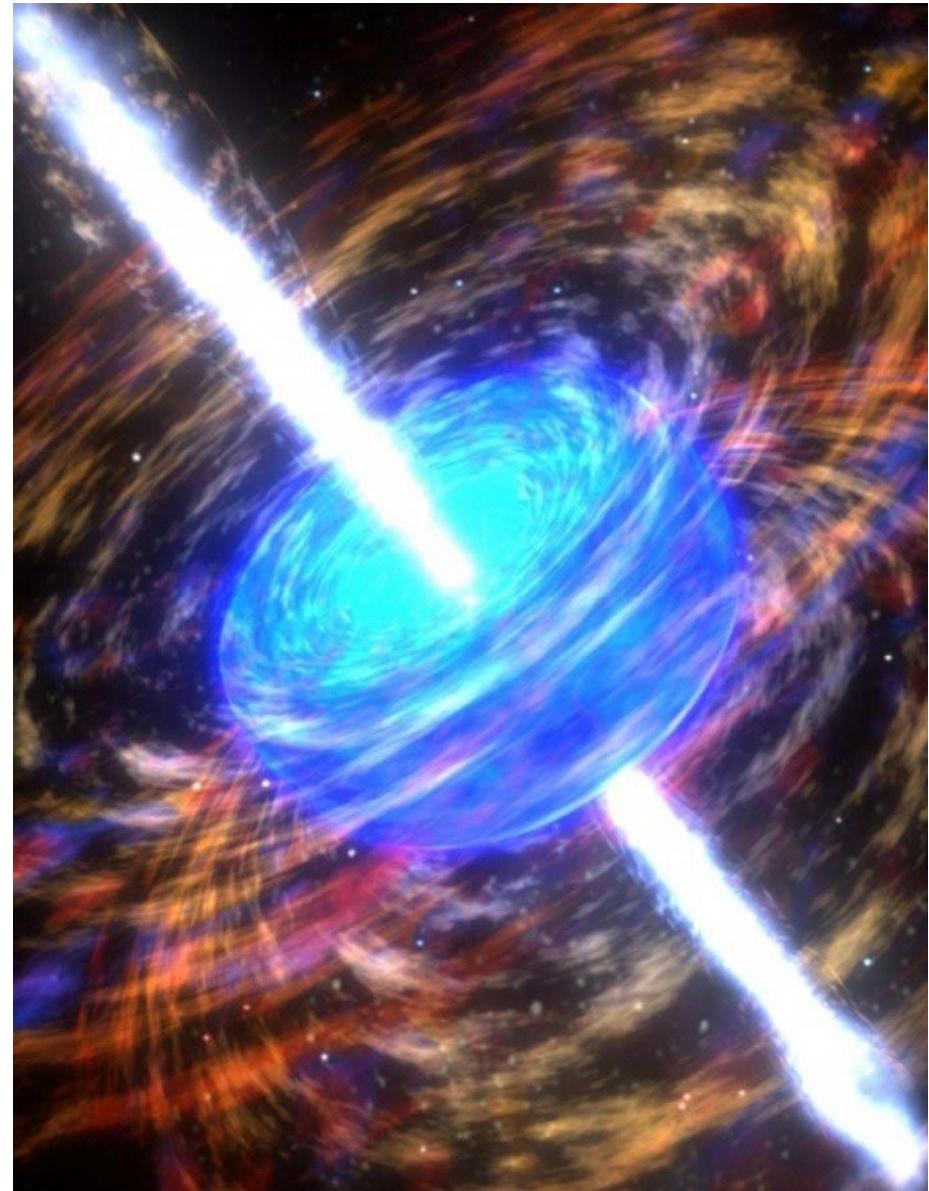
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The jets are common in space.

Non-relativistic jets:  
Young stars.

Relativistic jets:  
Micro quasars.  
Active Galactic Nuclei.  
Gamma Ray Bursts.

Lets make a stress on highly magnetized flow which should be in GRB case.



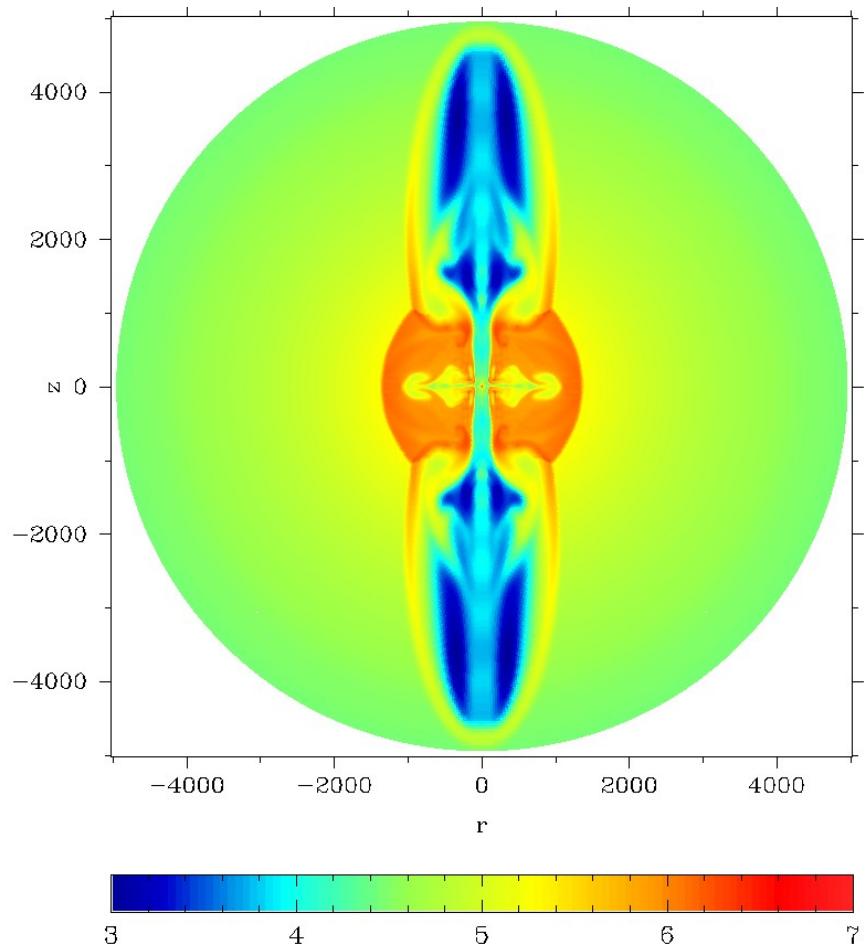
# GRB jets

Magnetar driven jet:

Energy extraction rate:  
Up to  $3 \times 10^{50}$  erg/s

Chemical decomposition:  
Baryon dominated

More details please find in  
MNRAS (2007) 382, 1029



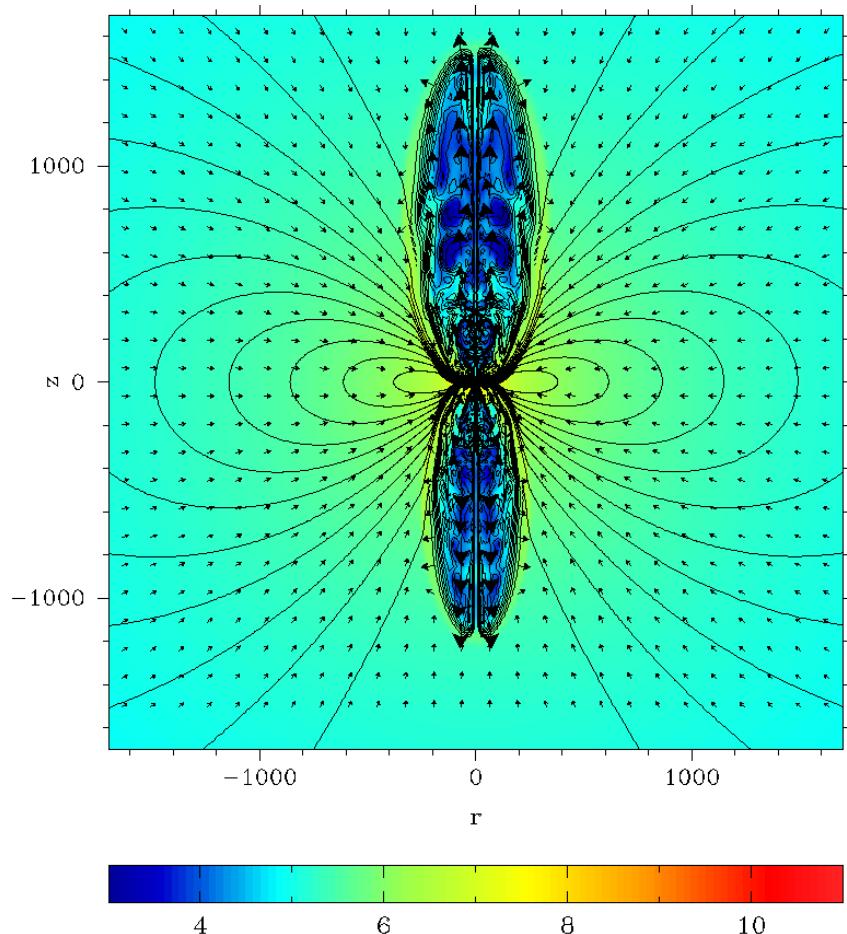
# GRB jets

Collapsar driven jet:

Energy extraction rate:  
Up to few  $10^{52}$  erg/s

Chemical decomposition:  
Lepton dominated

More details please find in  
MNRAS (2008) 385L, 28  
and arxiv:0809.1402



## Theoretical models:

Pure HD models do not provide height effective acceleration.

Acceleration due to radiation pressure does not allow to overcome limit  $\Gamma=2-5$ .

MHD approach allows acceleration up to high  $\Gamma$ .

Lovelace (1975) and Blandford (1976) first proposed the magnetically driven jet from accretion disks, and Blandford & Payne (1982) discussed magneto-centrifugally driven outflow from a Keplerian disk in steady, axisymmetric and self-similar situation.

Later a number of papers were dedicated to MHD acceleration of jets  
(Begelman, Beskin, Chiueh, Contopoulos, Konigl, Li, Nokhrina, Vlahakis, Bogovalov, Del Znna, Gammei, Koide, Shibata, Anton, Komissarov, Barkov)

Here we present a self-similar 2D ultrarelativistic MHD acceleration of jets.

## Basic Equations:

$$\left\{ \begin{array}{l} \frac{1}{c} \partial_t (\sqrt{-g} \rho u^t) + \partial_i (\sqrt{-g} \rho u^i) = 0, \\ \frac{1}{c} \partial_t (\sqrt{-g} T_v^t) + \partial_i (\sqrt{-g} T_v^i) = \frac{\sqrt{-g}}{2} \partial_v (g_{\alpha\beta}) T^{\alpha\beta}, \\ \frac{1}{c} \partial_t (B^i) + e^{ijk} \partial_j (E_k) = 0, \\ \partial_i (\sqrt{\gamma} B^i) = 0 \\ T^{\kappa v} = T_{(m)}^{\kappa v} + T_{(e)}^{\kappa v}, \\ T_{(m)}^{\kappa v} = w u^\kappa u^v / c^2 + p g^{\kappa v}, \\ T_{(e)}^{\kappa v} = \frac{1}{4\pi} \left[ F^{\kappa\alpha} F_\alpha^v - \frac{1}{4} (F^{\alpha\beta} F_{\alpha\beta}) g^{\kappa v} \right], \\ B^i = \frac{1}{2} e^{ijk} F_{jk}, \quad E_i = -e_{ijk} \frac{u^j}{u^t} \frac{B^k}{c}, \\ p = Q \rho^{4/3}. \end{array} \right.$$

To maintain a firm control over the jet's confinement and to prevent complications related to numerical diffusion of the dense nonrelativistic plasma from the jet's surroundings, we study outflows that propagate inside a solid funnel of a prescribed shape. Specifically, we consider axisymmetric paraboloidal funnels

$$z \propto r^a, \quad a = 2/3, 1, 3/2, 2, 3$$

we employ elliptical coordinates  $\{\xi, \eta, \varphi\}$ , where

$$\xi = rz^{-1/a}$$

and

$$\eta^2 = \frac{r^2}{a} + z^2$$

We use a Godunov-type numerical code based on the scheme described in (Komissarov 1999). To reduce numerical diffusion we applied parabolic reconstruction instead of the linear one of the original code. The code is MPI parallelized.

## High magnetization models:

Model	a	rotation	w/ $\rho_0 c^2$	$\theta_i$ or $\xi_i$	$\mu_{max}$
A	1	uniform	1.0	$\theta_i = 0.2$	560
AW	1	uniform	1.0	$\theta_i = \pi/2$	560
B1	3/2	uniform	1.0	$\xi_i = 2.0$	620
B2	3/2	uniform	1.0	$\xi_i = 2.0$	310
B2H	3/2	uniform	55	$\xi_i = 2.0$	370
B3	3/2	uniform	1.0	$\xi_i = 2.0$	155
B4	3/2	uniform	1.0	$\xi_i = 2.0$	78
B5	3/2	uniform	1.0	$\xi_i = 2.0$	39
C	2	uniform	1.0	$\xi_i = 2.0$	620
D	3/2	differential	1.0	$\xi_i = 2.0$	600
E	2/3	uniform	1.0	$\xi_i = 0.1$	300
F	3	uniform	1.0	$\xi_i = 2.0$	540

$$\Omega = \Omega_0 \left[ 1 + 0.778 \left( \frac{\xi}{\xi_{max}} \right)^2 - 1.778 \left( \frac{\xi}{\xi_{max}} \right)^3 \right] \quad OR \quad \Omega = \Omega_0$$

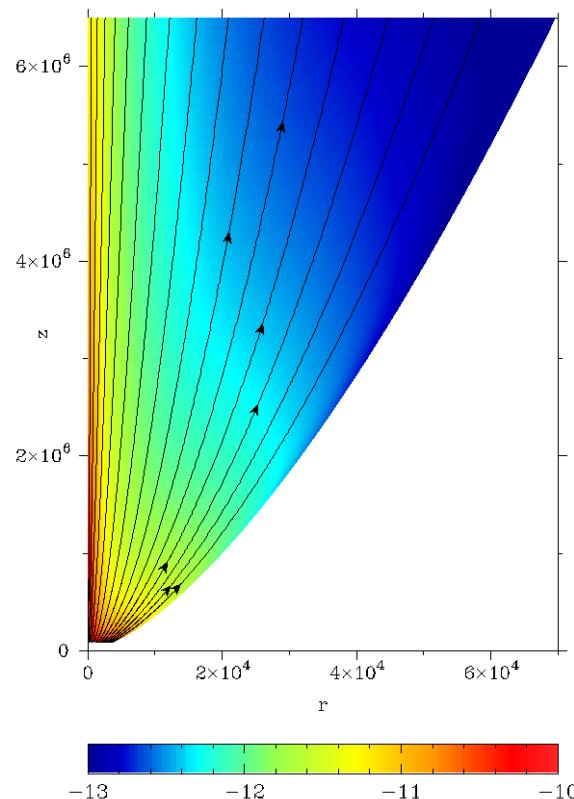
$$\mu = \Gamma(1 + \sigma)$$

## Non uniform rotation

$$\Omega = \Omega_0 \left[ 1 + 0.778 \left( \frac{\xi}{\xi_{\max}} \right)^2 - 1.778 \left( \frac{\xi}{\xi_{\max}} \right)^3 \right]$$

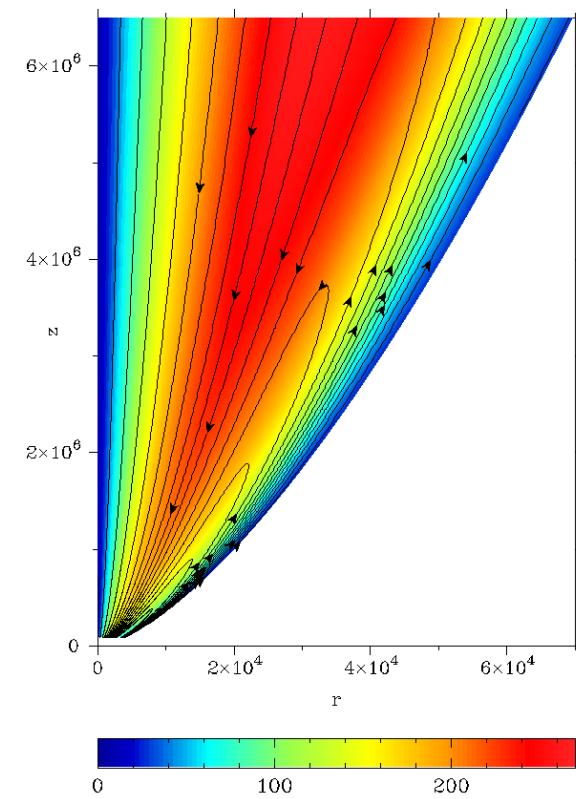
$$z \propto r^{3/2}$$

$$\log_{10}(\rho)$$



$$\mu = \Gamma (1 + \sigma) \approx 600$$

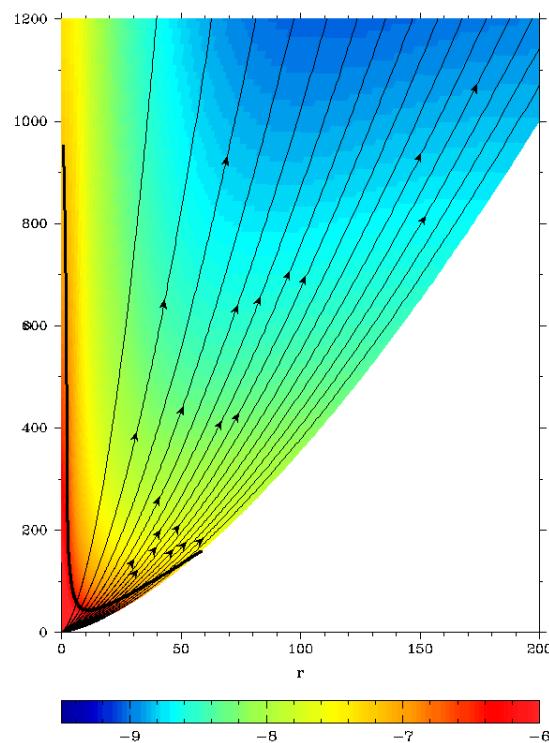
$$\Gamma$$



## Uniform rotation

$$z \propto r^{3/2}$$

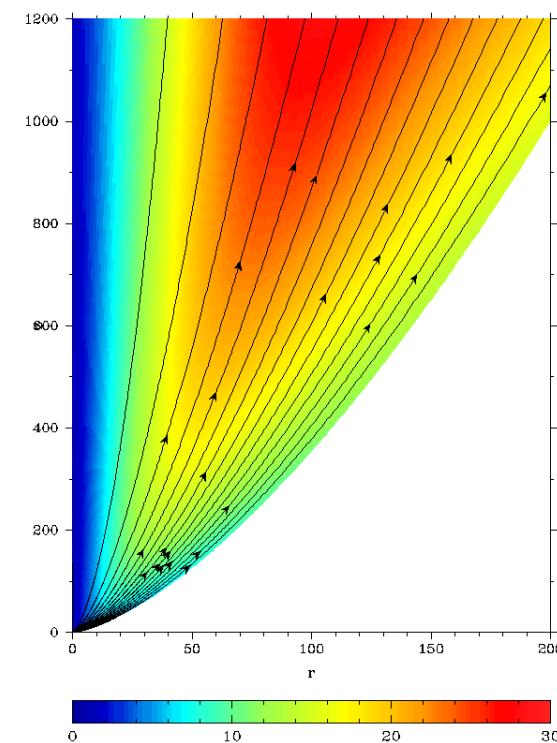
$$\log_{10}(\rho)$$



$$\Omega = \Omega_0$$

$$\mu = \Gamma(1 + \sigma) \approx 620$$

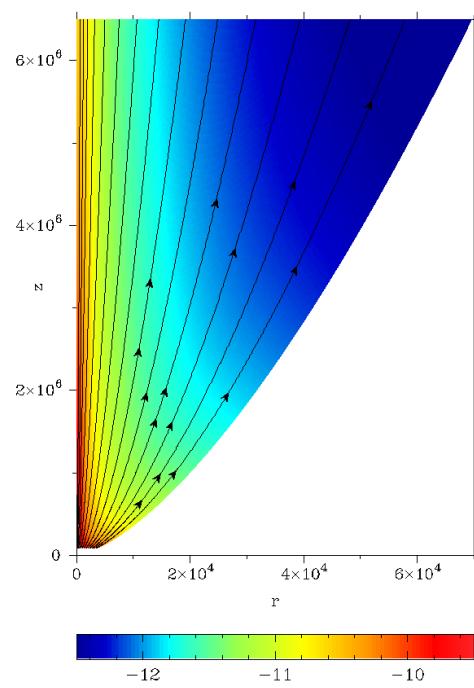
$$\Gamma$$



## Uniform rotation

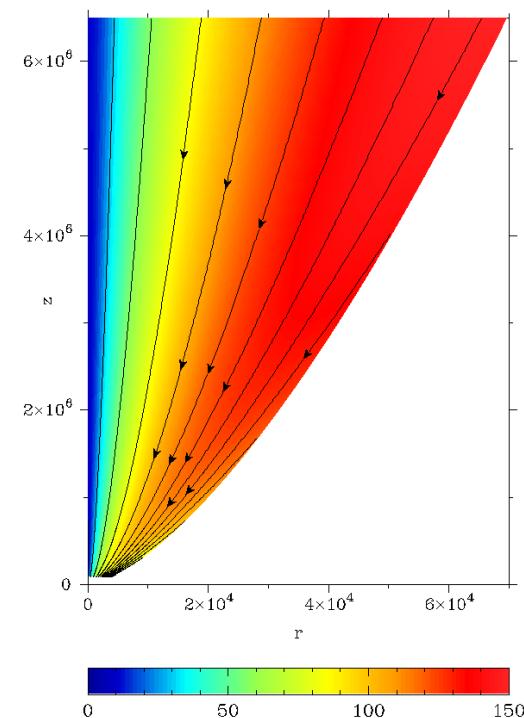
$$z \propto r^{3/2}$$

$$\log_{10}(\rho)$$



$$\Omega = \Omega_0$$
$$\mu = \Gamma(1 + \sigma) \approx 620$$

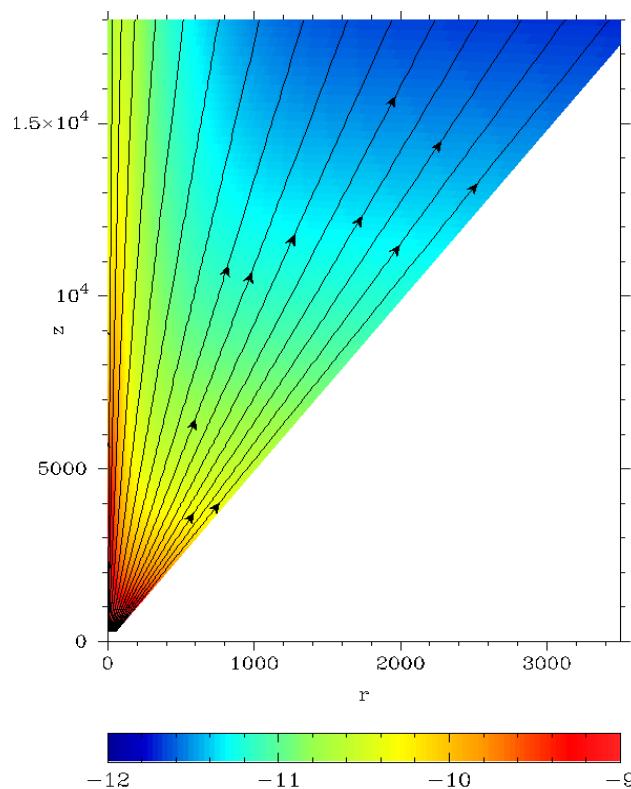
$$\Gamma$$



## Uniform rotation

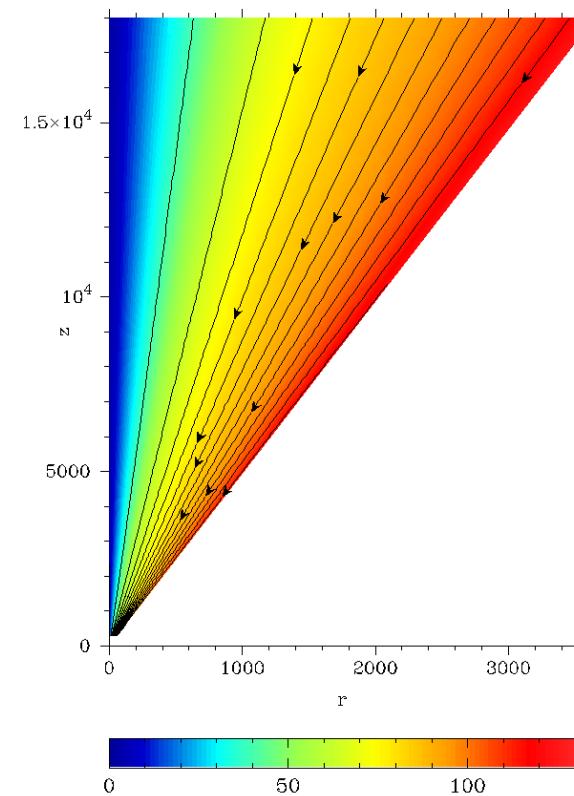
$$z \propto r$$

$$\log_{10}(\rho)$$



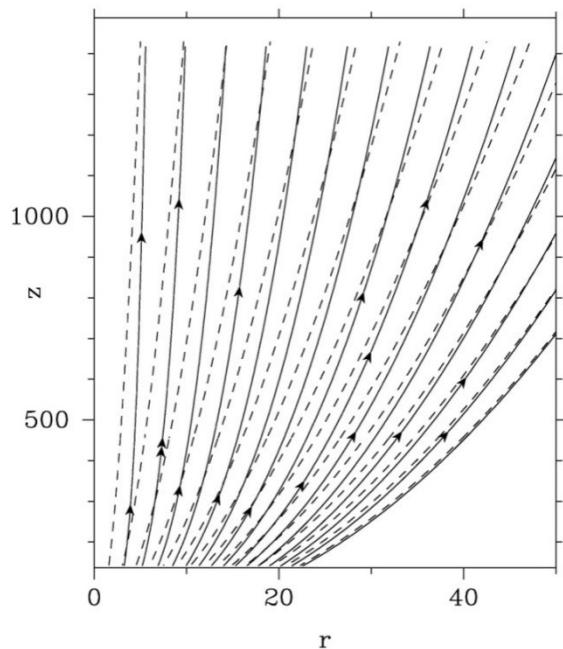
$$\Omega = \Omega_0$$
$$\mu = \Gamma(1 + \sigma) \approx 560$$

$$\Gamma$$

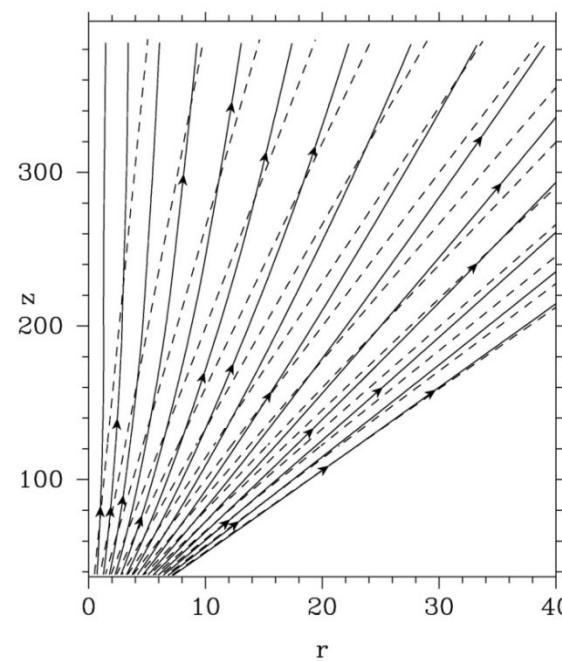


## Self collimation of magnetic lines

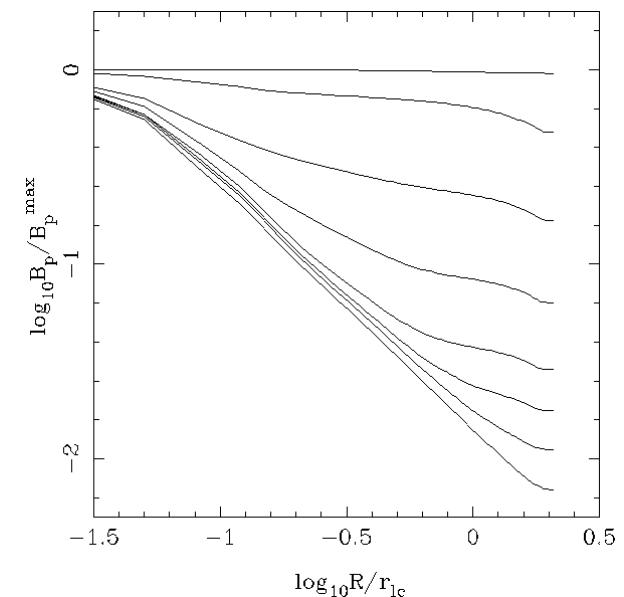
$$z \propto r^2$$



$$z \propto r^1$$



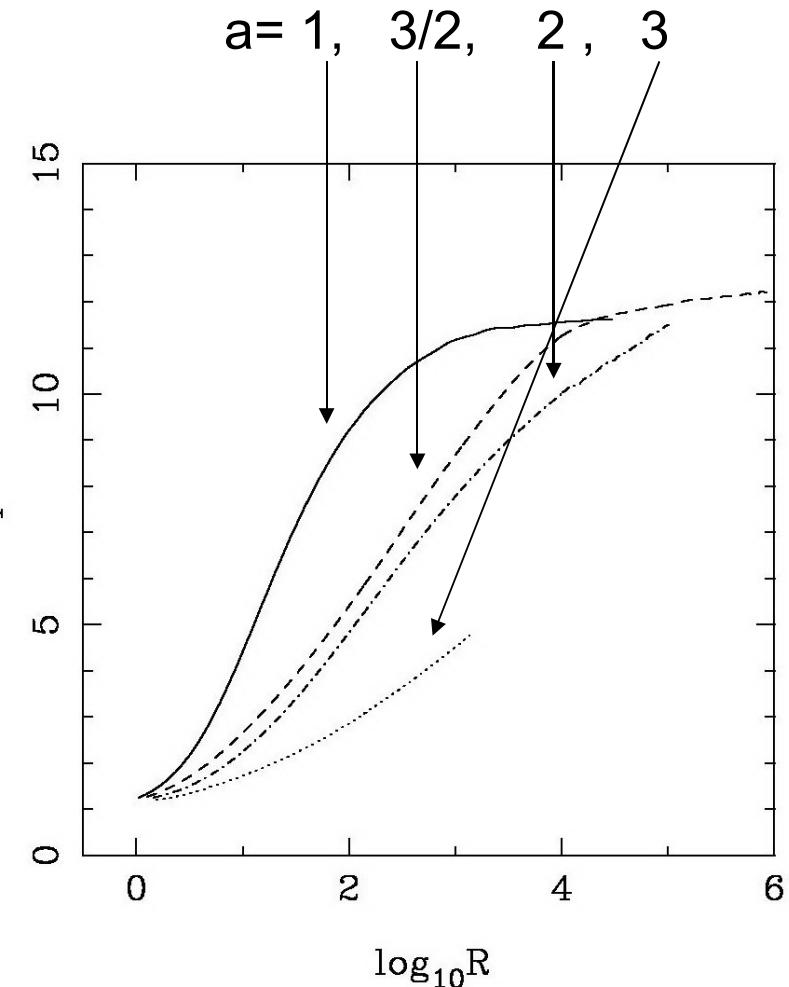
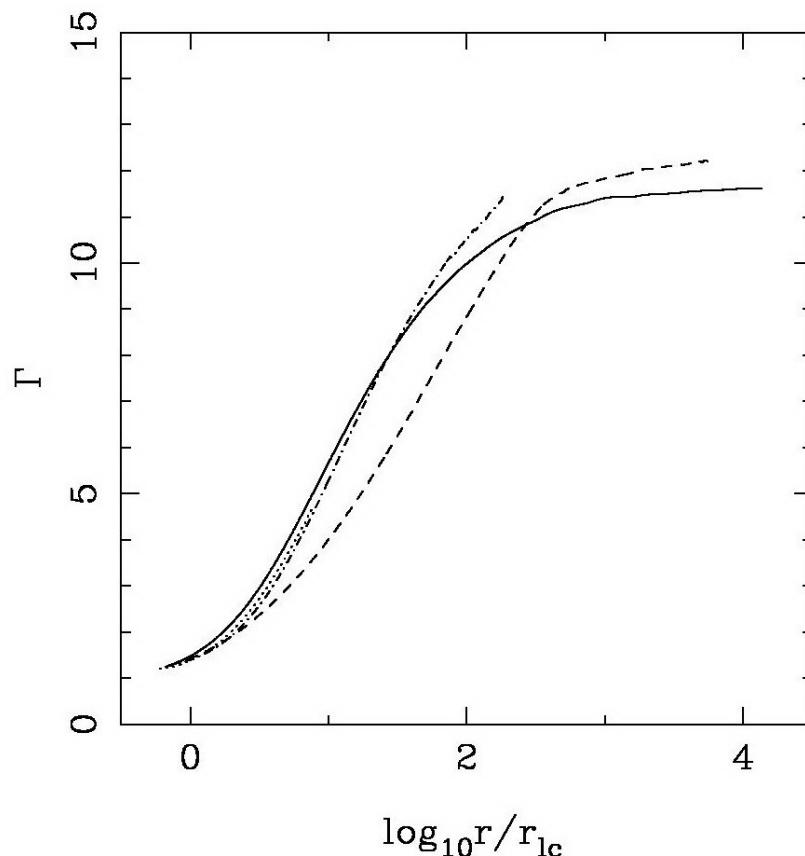
Distribution of poloidal magnetic field across the jet showing the development of axial core



Acceleration rate depends of geometry:

$$Z \propto r^a$$

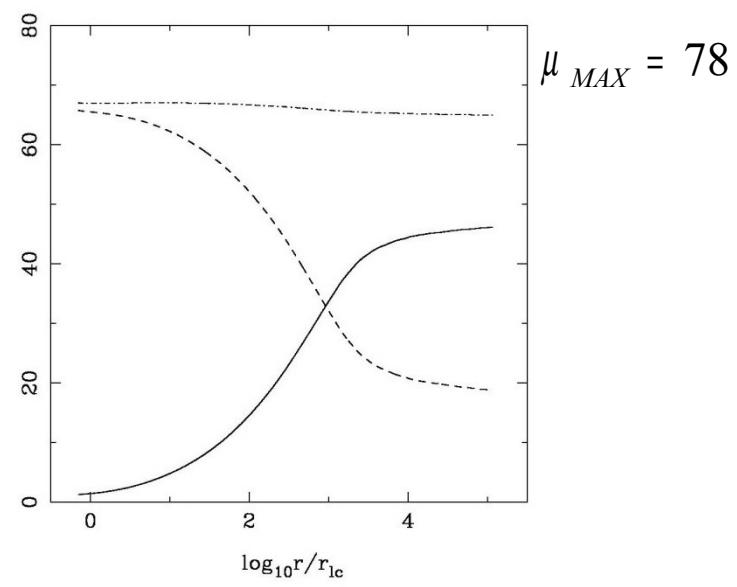
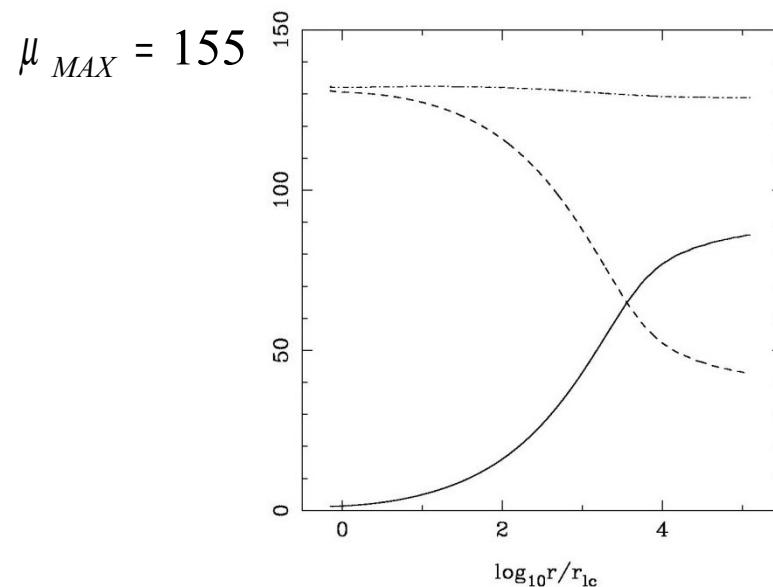
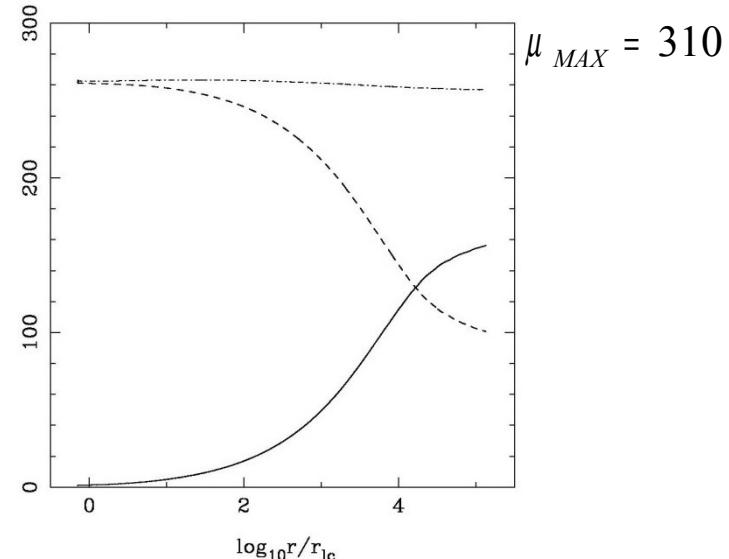
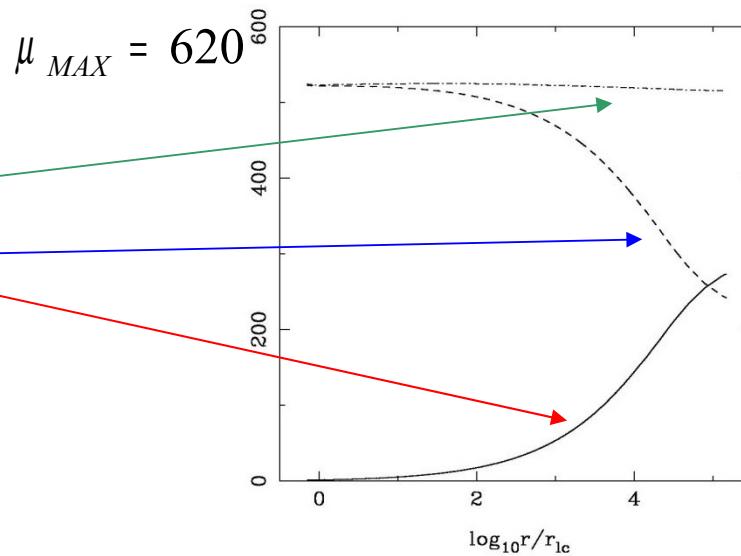
$$\Omega = \Omega_0$$





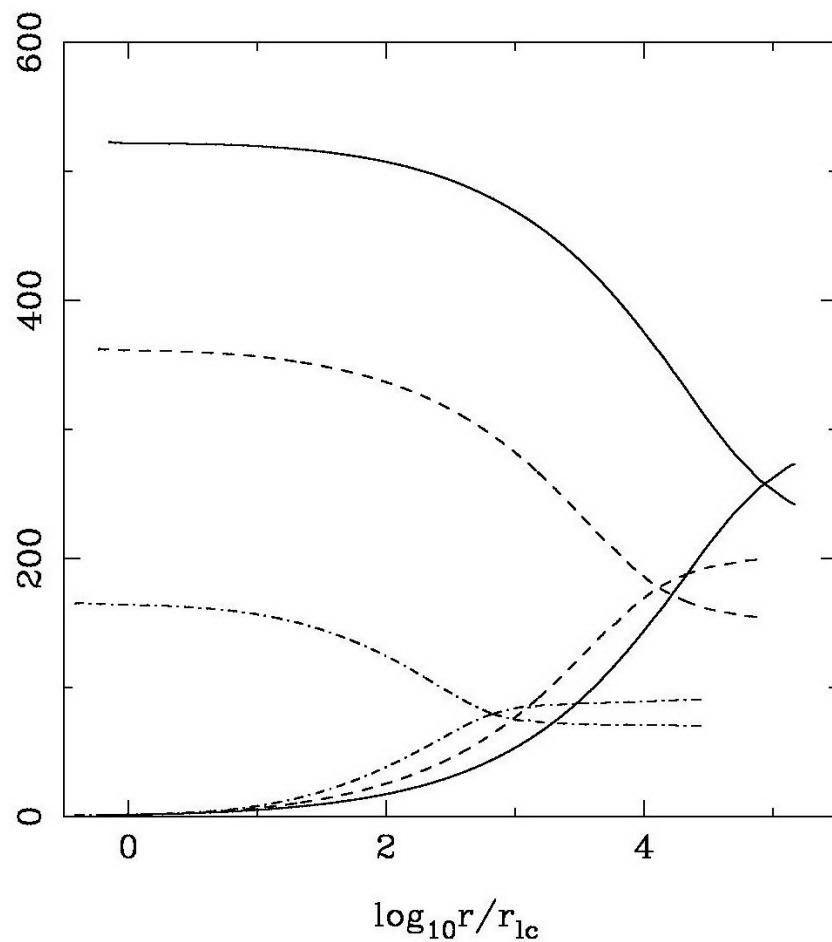
$$\mu = \Gamma(1 + \sigma)$$

$$Z \propto r^{3/2}$$

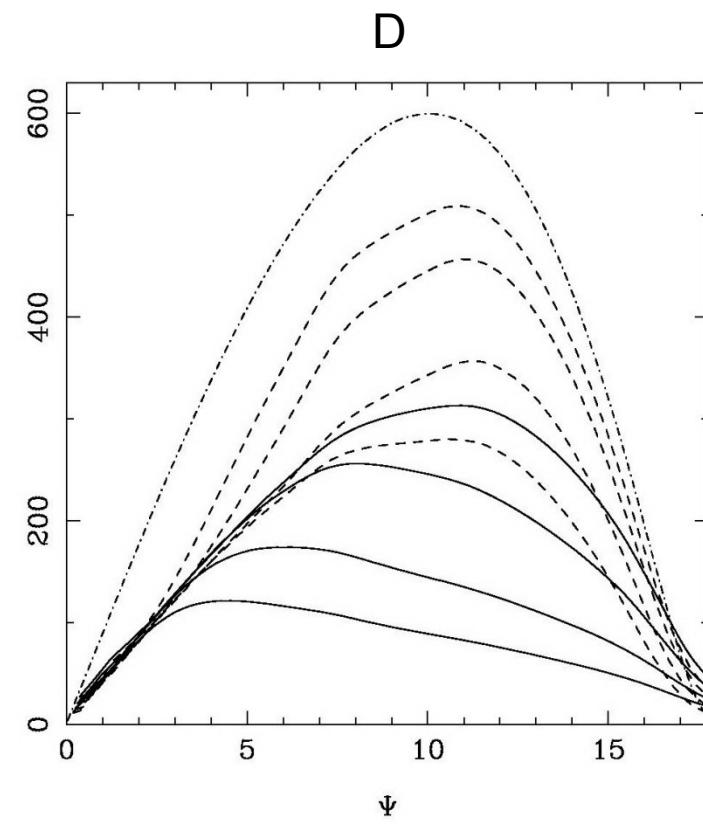
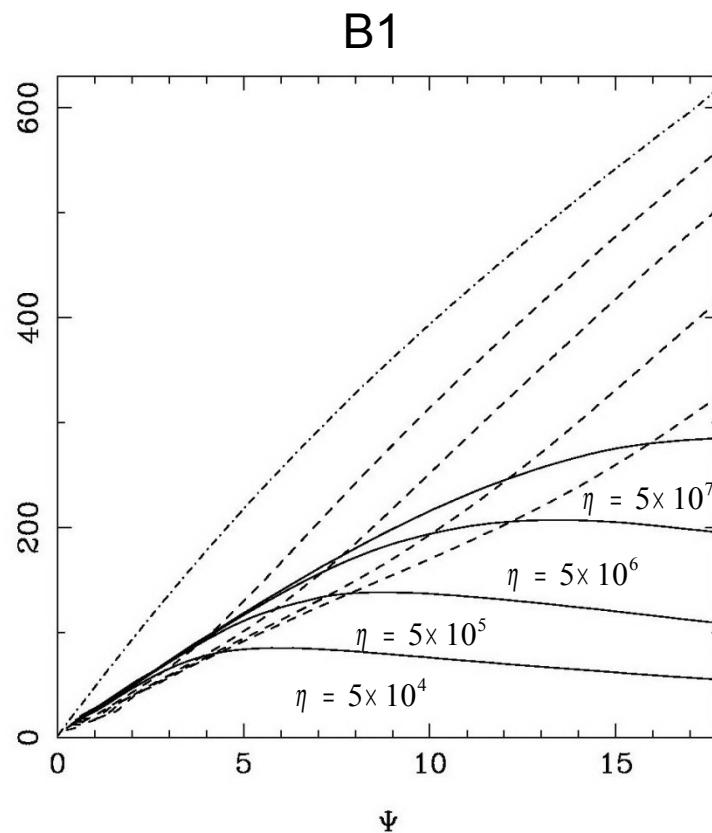




$\Gamma$  (lower branch)  
and  $\sigma\Gamma$  (upper branch) along the  
magnetic field line  $\Psi=0.8 \Psi_{\max}$   
(solid lines),  
along the magnetic field line  
 $\Psi=0.5 \Psi_{\max}$  (dashed lines), and  
along the magnetic field line  
 $\Psi=0.2 \Psi_{\max}$  (dash-dotted lines)  
in model B1.



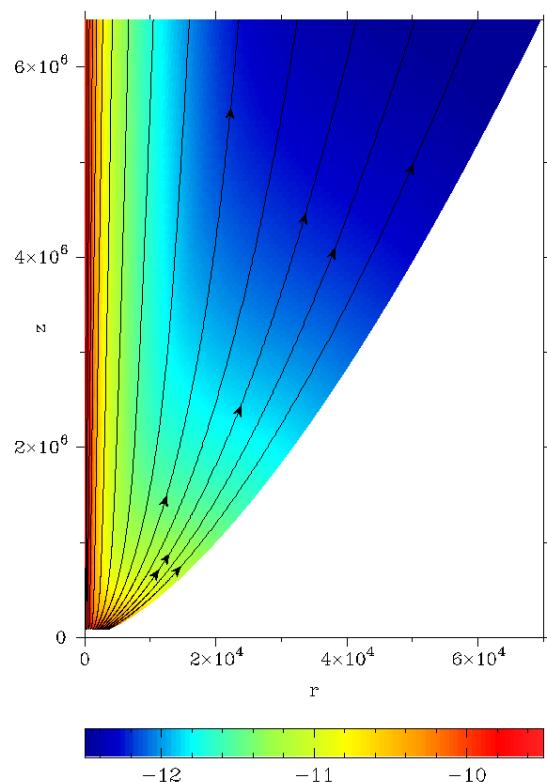
Distribution of  $\Gamma$  and  $\sigma\Gamma$  across the jet. Solid lines show  $\Gamma$ , dashed lines show  $\sigma\Gamma$  and the dash-dotted line shows  $\mu$ .



## Uniform rotation. Hot plasma

$$z \propto r^{3/2}$$

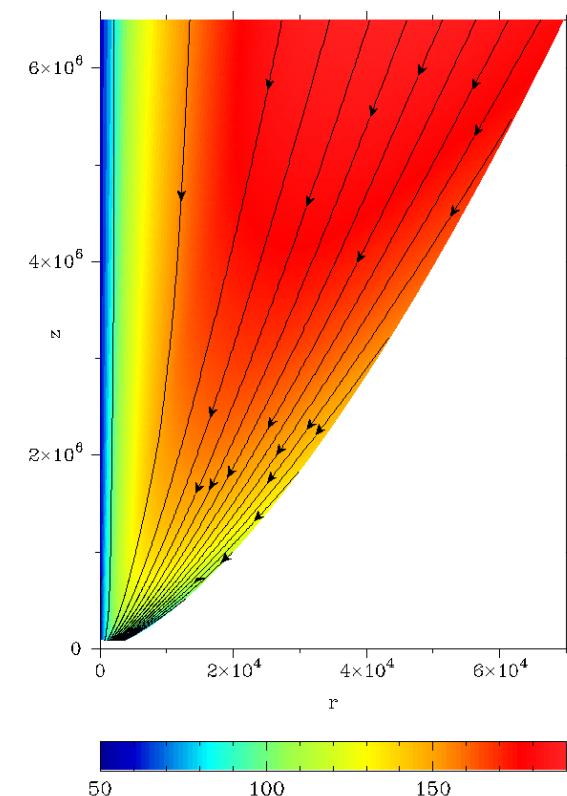
$$\log_{10}(\rho)$$



$$\Omega = \Omega_0$$

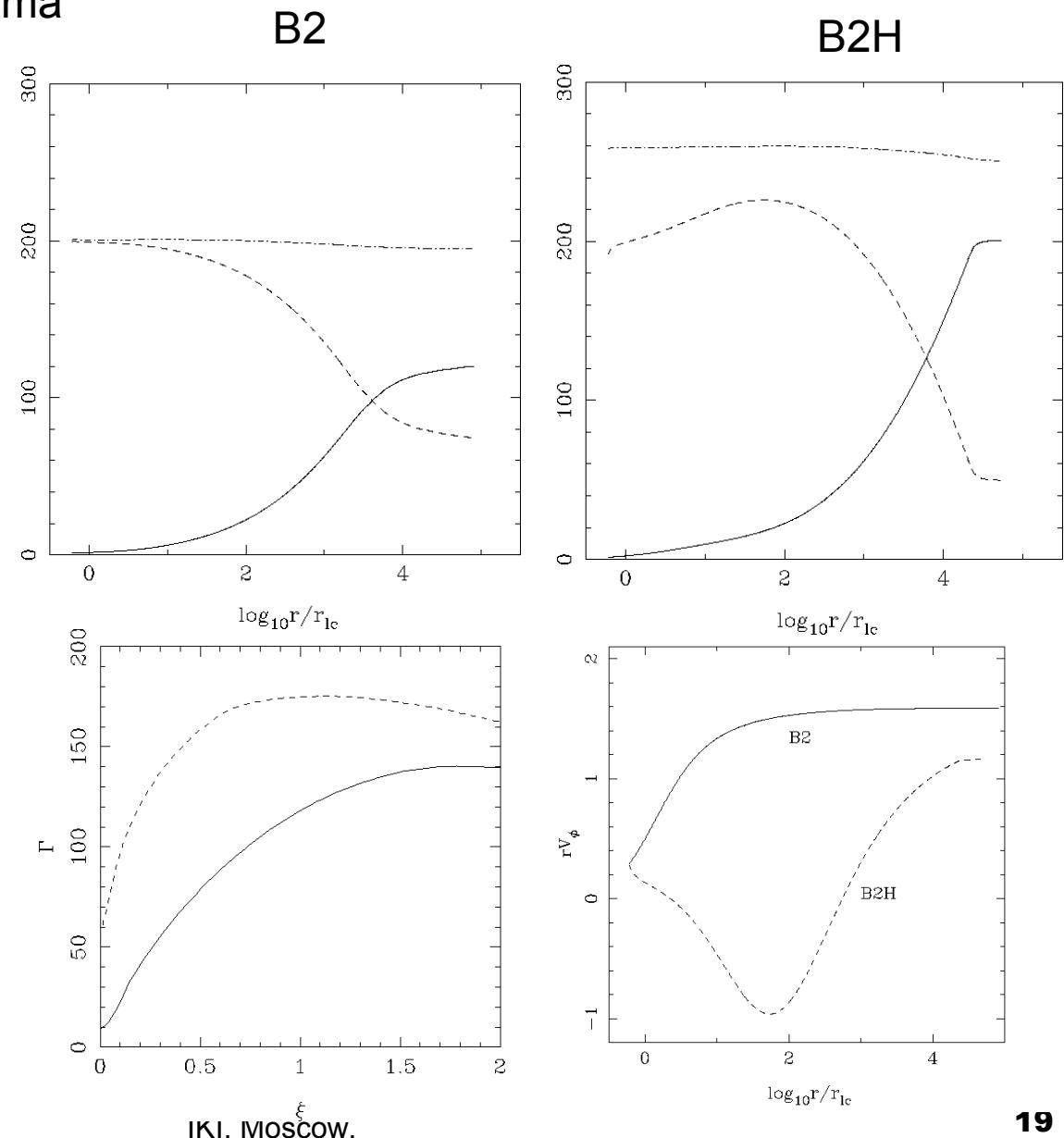
$$\mu = \Gamma(1 + \sigma) \approx 620$$

$$\Gamma$$



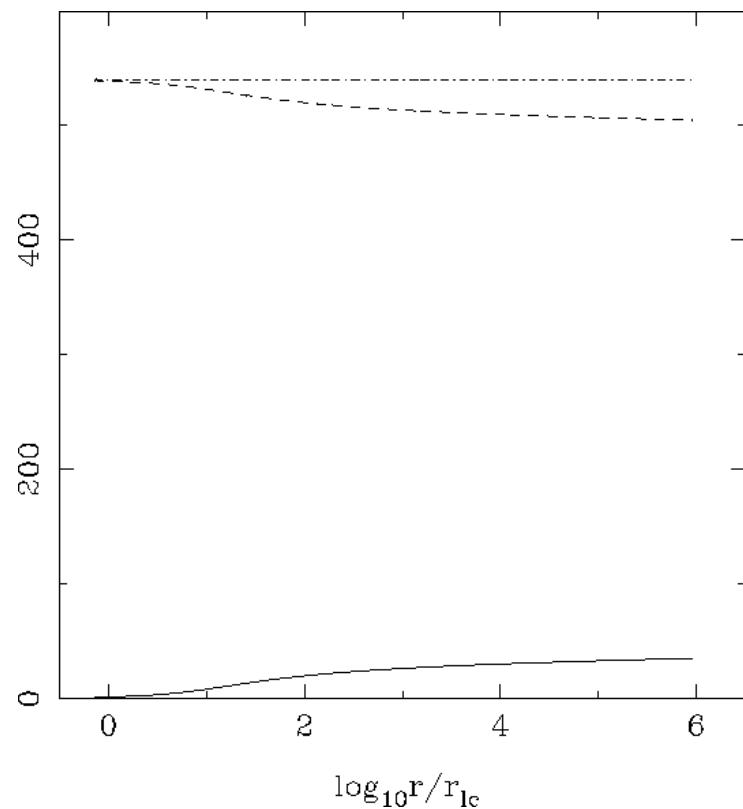
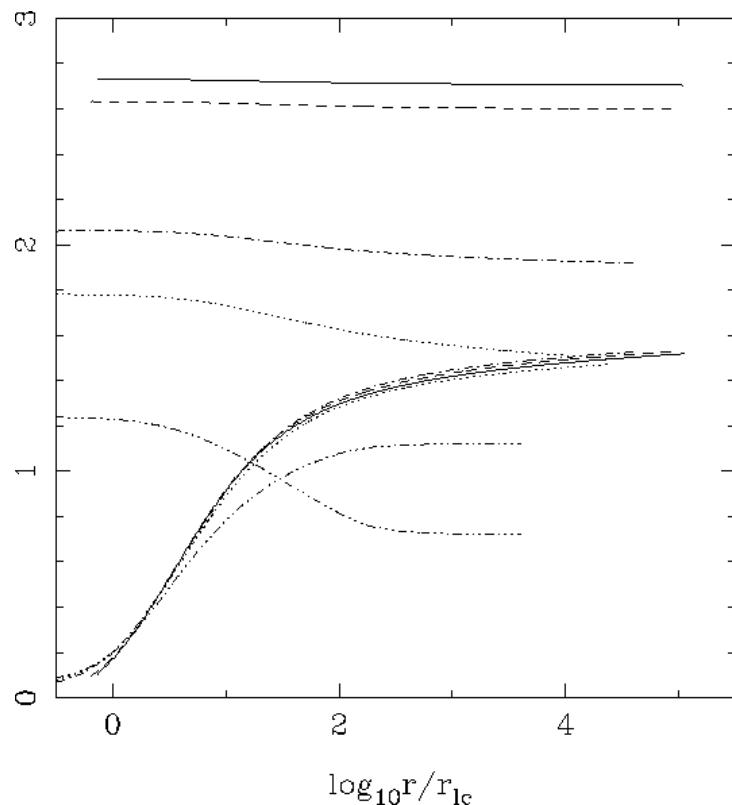
# Uniform rotation. Hot plasma

Initially thermal energy transferred to magnetic one and after that magnetic energy converted to kinetic balk motion



Unconfined wind solution (model AW).  
Lorentz factor (increasing functions)  
and  $\Gamma \sigma$  (decreasing function)

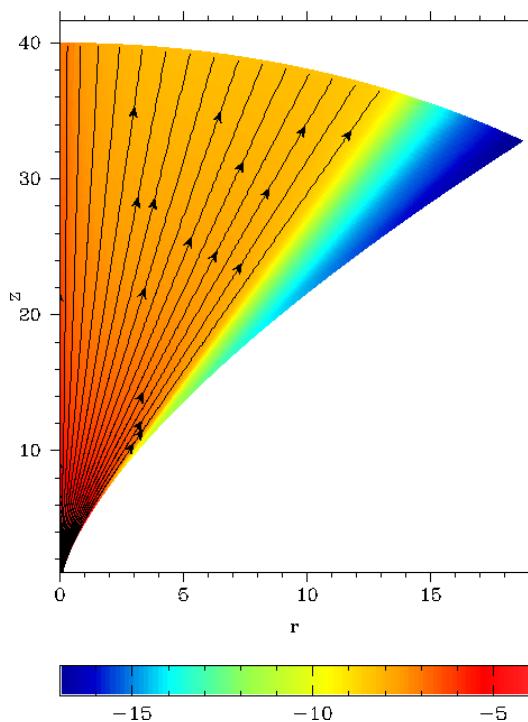
Ineffective



## Uniform rotation

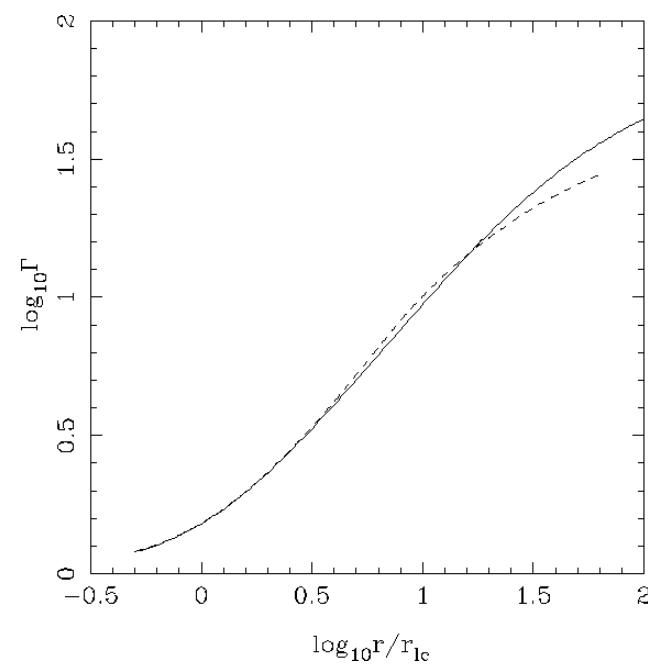
$$z \propto r^{2/3} \quad \Omega = \Omega_0 \quad \mu = \Gamma(1 + \sigma) \approx 300$$

$\log_{10}(\rho)$

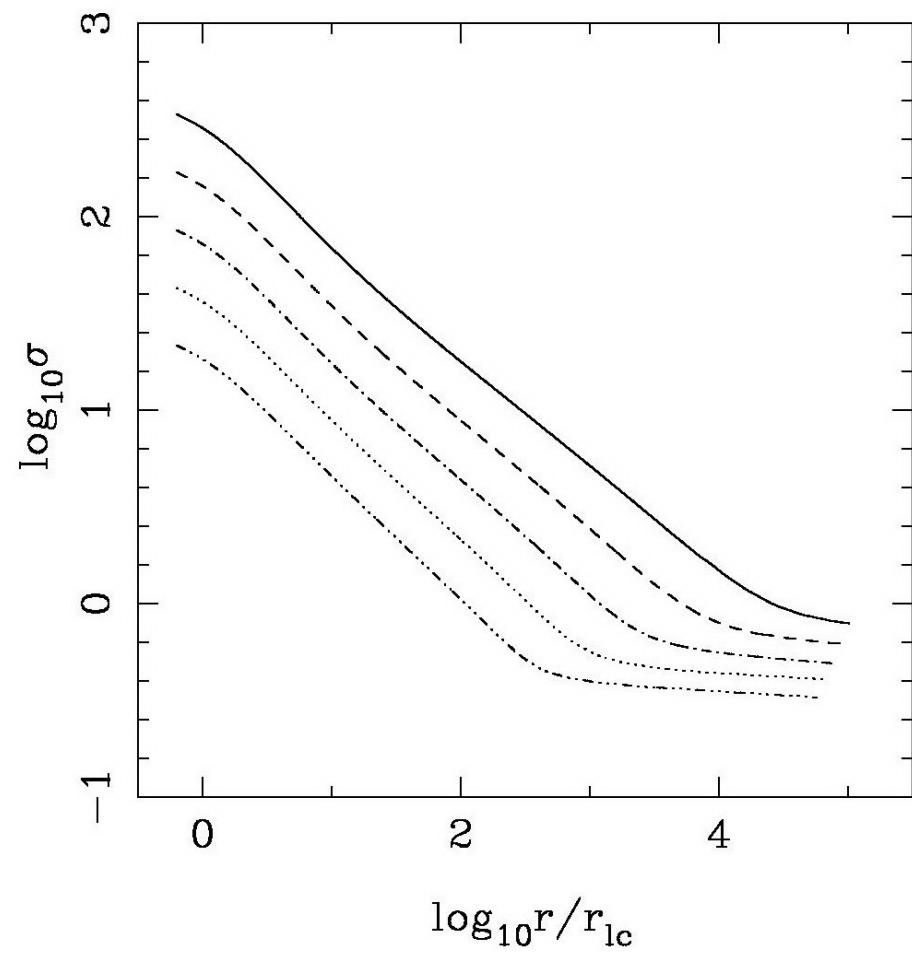


That happens then jet leave the star envelop?

It takes conical shape!



Dependence of magnetization  
from cylindrical radius for set  
of maximum magnetization.



$$b = 3/2$$

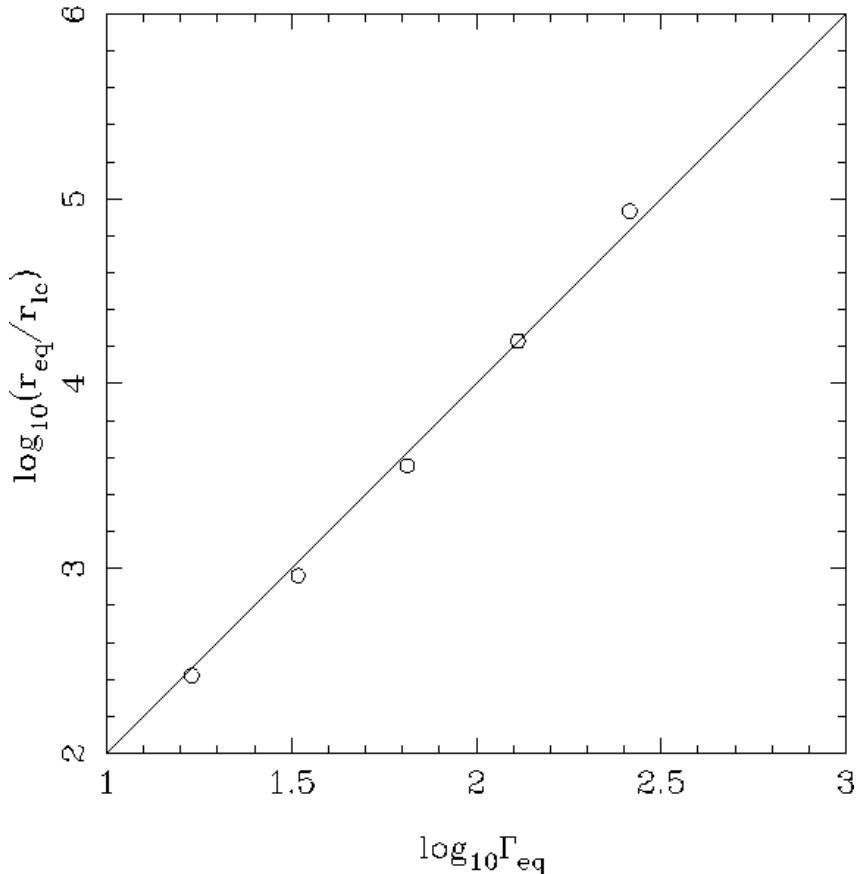
Dependence of ‘equilibrium’ radius  $r_{eq}$  from maximal magnetization  $\Gamma_{eq} = \frac{\mu}{2}$

$r_{eq}$  is the radius there  $\sigma \equiv 1$  or half of magnetic energy transferred to kinetic energy of matter.

$$r_{eq} \approx r_{lc} \left( \frac{\mu}{2} \right)^{1/(b-1)}$$

$$R_{eq} \approx r_{lc} \left( \frac{\mu}{2} \right)^{b/(b-1)}$$

$r_{eq}$  - cylindrical  
 $R_{eq}$  - spherical



radius of equipartition between Poynting and matter energy fluxes

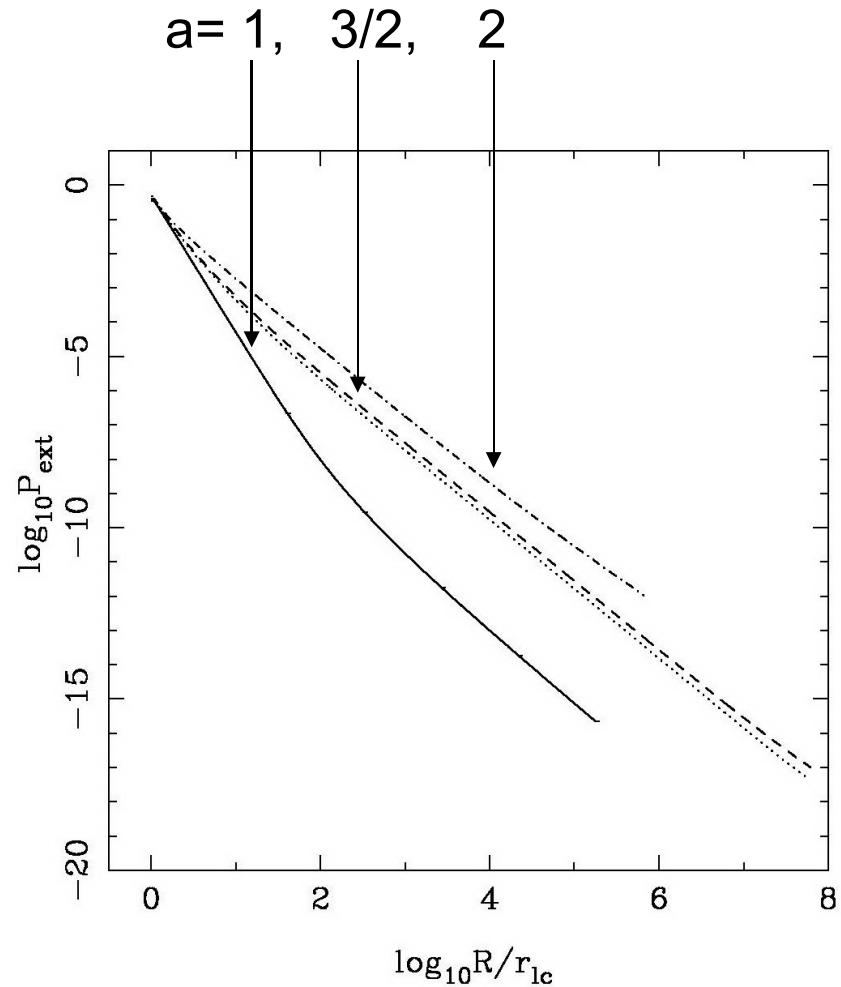
$$z \propto r^\alpha \quad \mu_{MAX} = 620$$

The effective pressure deduction as a function of spherical radius.  
We can calculate mean power-low indices  $p_{ext} = R^{-\alpha}$

a	1	3/2	2
$\alpha$	4.6÷2.4	2	1.85

It is difficult to distinguish cases with  $a > 1.5$ .

For example, in a spherical wind of polytropic index 5/3, the thermal pressure scales as  $R^{-10/3}$  and the ram pressure as  $R^{-5/2}$ . Thus, a disk wind that assumes a nearly spherical geometry not too far from the origin could effectively confine a relativistic jet with a nearly conical outer boundary.

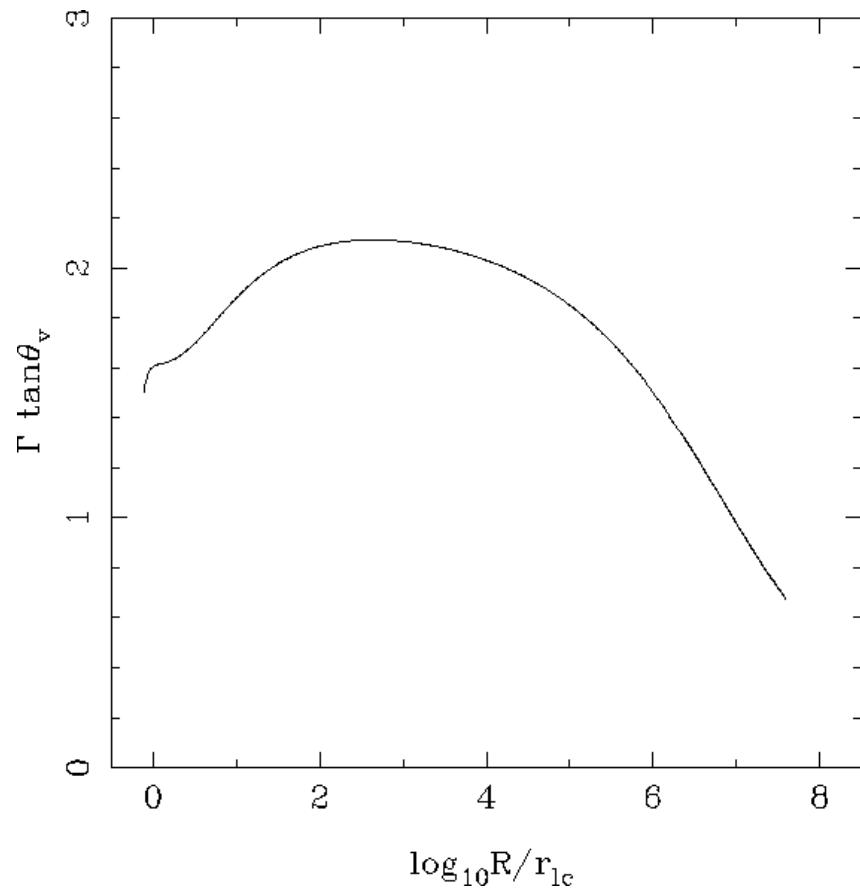


Jet half-opening angle of the magnetic flux surface

$$\Gamma \tan \theta \approx \frac{1}{\sqrt{b - 1}}$$

if

$$1 < b \leq 2$$



## Application for GRB jets

Magnetic field geometry:  $z=r^b$

$$1 \leq b \leq 2$$

$$r_{lc} \approx 4r_g = 6 \times 10^5 \left( \frac{M}{M_{sun}} \right) cm$$

$$R_\gamma \approx \Gamma^2 c \delta t = 3 \times 10^{13} \left( \frac{\Gamma}{100} \right)^2 \left( \frac{\delta t}{0.1 s} \right) cm$$

$$\dot{M}_j \approx 5 \times 10^{-7} \left( \frac{E}{10^{51} erg} \right) \left( \frac{\Gamma}{100} \right)^{-1} \left( \frac{\Delta t}{10 s} \right)^{-1} M_{sun} s^{-1}$$

$b=3/2$  or  $b=3$

$$R \approx 10^{12} \left( \frac{\Gamma}{100} \right)^3 cm$$

$$\mu < 430 \left( \frac{R_\gamma}{10^{13} cm} \right)^{1/3}$$

$b=2$

$$R \approx 10^{10} \left( \frac{\Gamma}{100} \right)^2 cm$$

$$\mu < 6 \times 10^3 \left( \frac{R_\gamma}{10^{13} cm} \right)^{1/2}$$

Where:

$$r_g \equiv \frac{GM}{c^2}$$

For short GRBs:

$$\Gamma < 30$$

$$R_{wind} \approx 3 \times 10^9 \left( \frac{v_{wind}}{0.1 c} \right) \left( \frac{\Delta t}{1 s} \right) cm$$

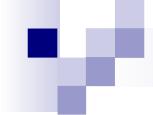
$$\Gamma < 100$$

# Conclusions:

- We have got highly ( $>50\%$ ) effective mechanism of jet acceleration.
- The acceleration is extended ( $r_{eq}$  order  $\mu^2 r_{lc}$ )
- Good agreement with AGN jets observations was achieved.
- Due to self collimation jet very narrow.  $\theta \approx \frac{2}{\Gamma}$

More details you can find in:

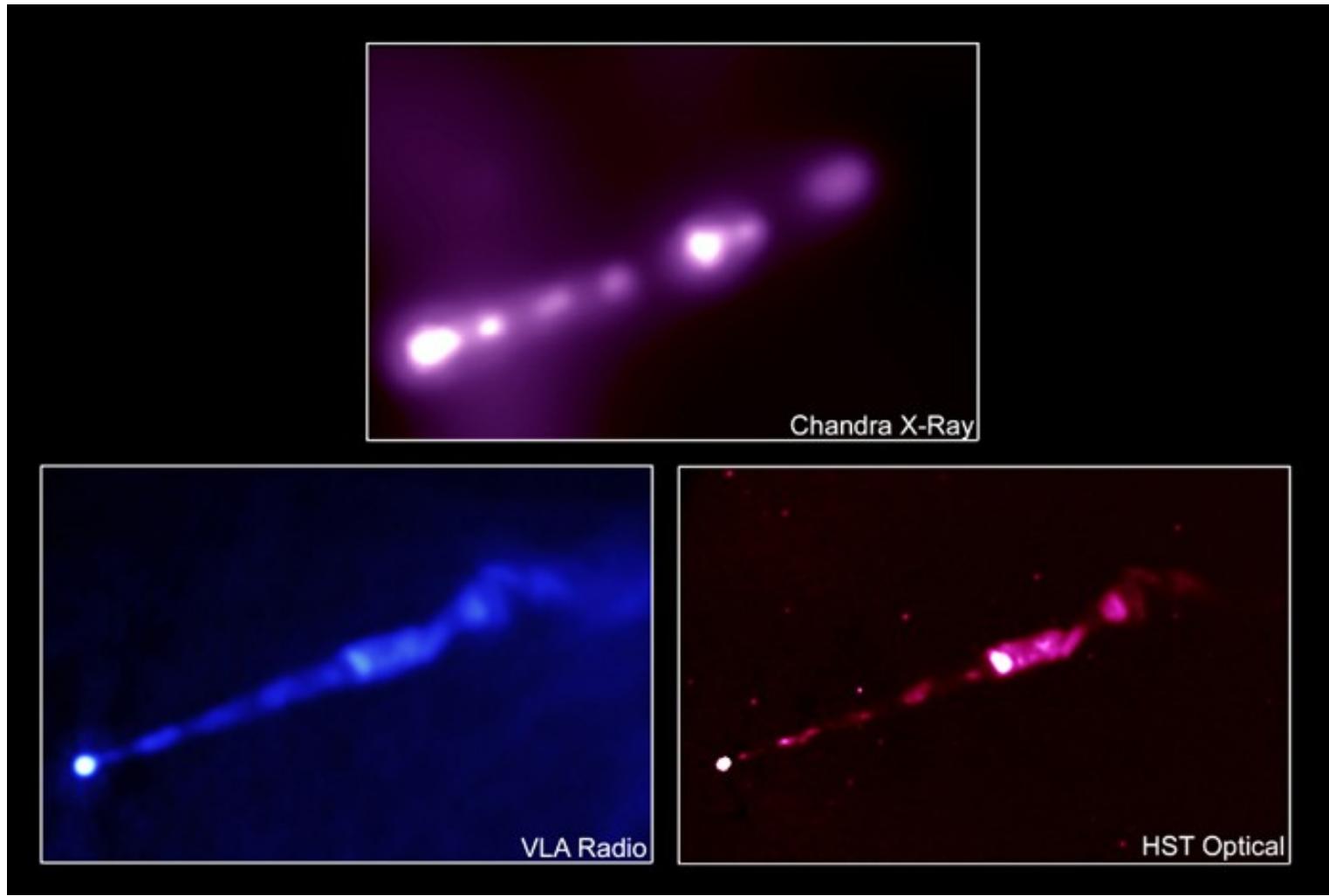
MNRAS 380, 51 ([astro-ph/0703146](#)),  
IJMP D 17, 10, 1669 ([arXiv:0801.4861](#)),  
MNRAS accepted ([arXiv:0811.1467](#))



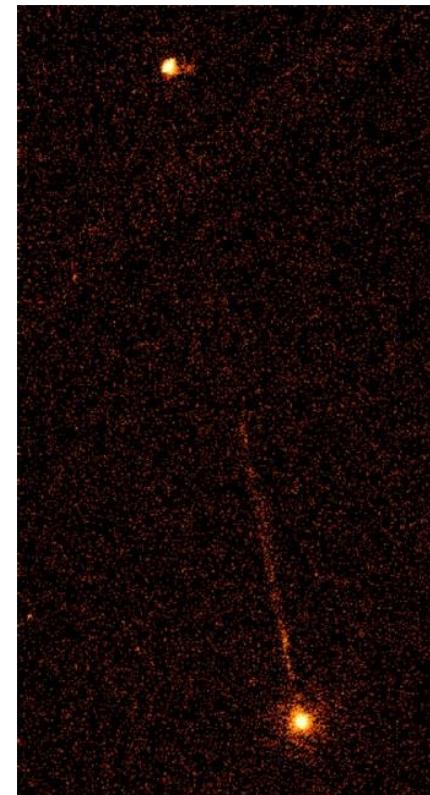
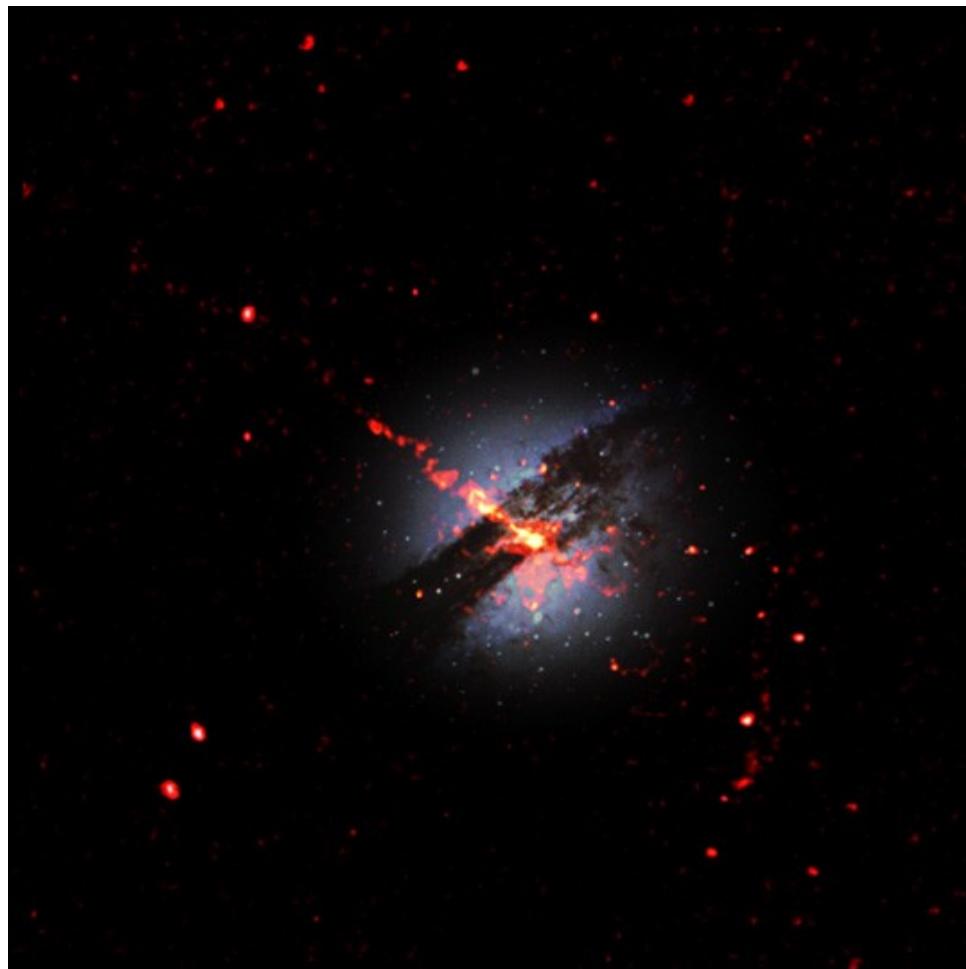
Thank you for yours attention!

The End!

## An energetic jet from the core of giant elliptical galaxy M87



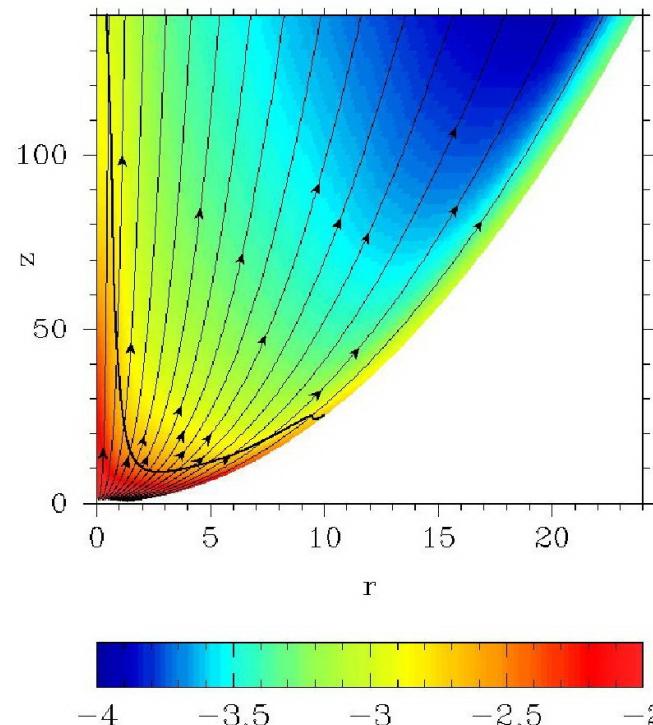
The Lorentz factors of blazar jets lie in the range  $\approx 5\text{--}40$ , with the majority of quasar components having  $\Gamma \approx 16\text{--}18$  and with BL Lac objects possessing a more uniform  $\Gamma$  distribution. The length of the jet lays  $0.1\text{pc} - 1\text{ Mpc}$ .



## Non-uniform rotation

$$z \propto r^2$$

$$\log_{10}(\rho)$$

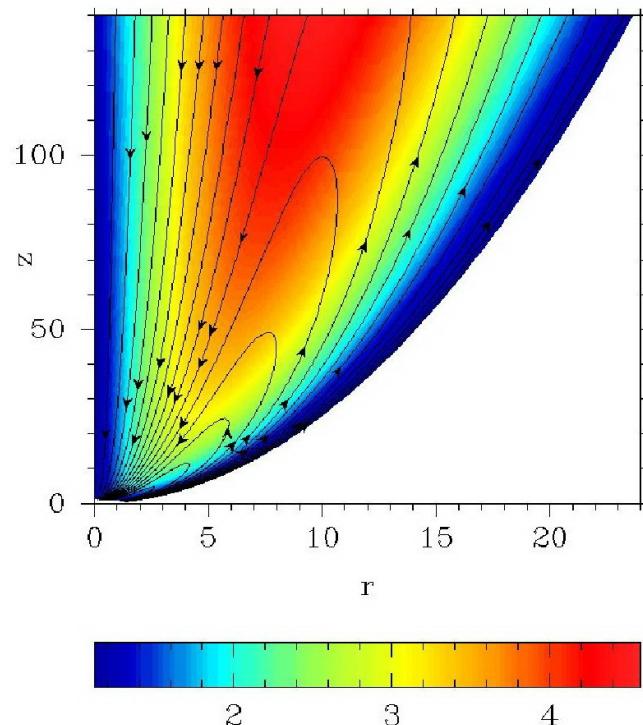


$$\Omega = \Omega_0 \left[ 1 + 0.778 \left( \frac{\xi}{\xi_{\max}} \right)^2 - 1.778 \left( \frac{\xi}{\xi_{\max}} \right)^3 \right]$$

$$\mu = \Gamma(1 + \sigma) \approx 11$$

$T=400$

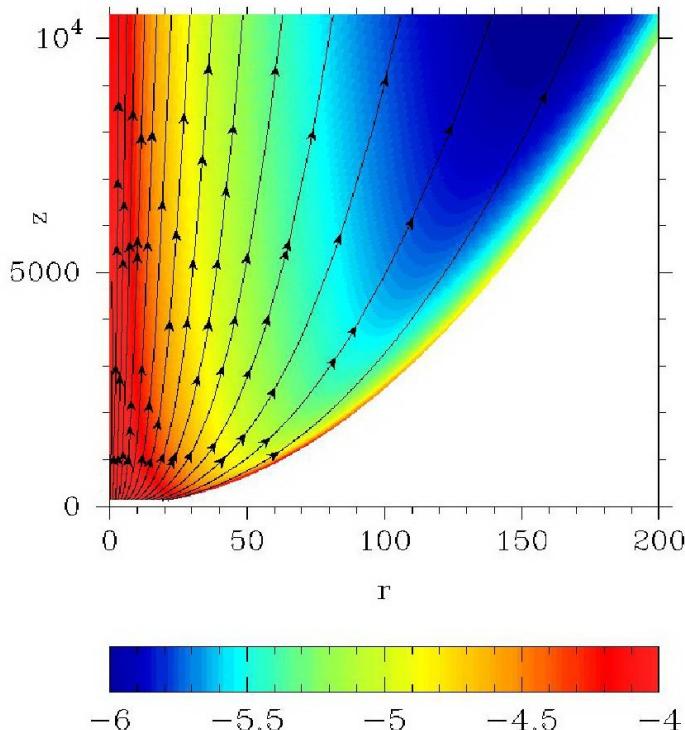
$\Gamma$



## Non-uniform rotation

$$z \propto r^2$$

$$\log_{10}(\rho)$$

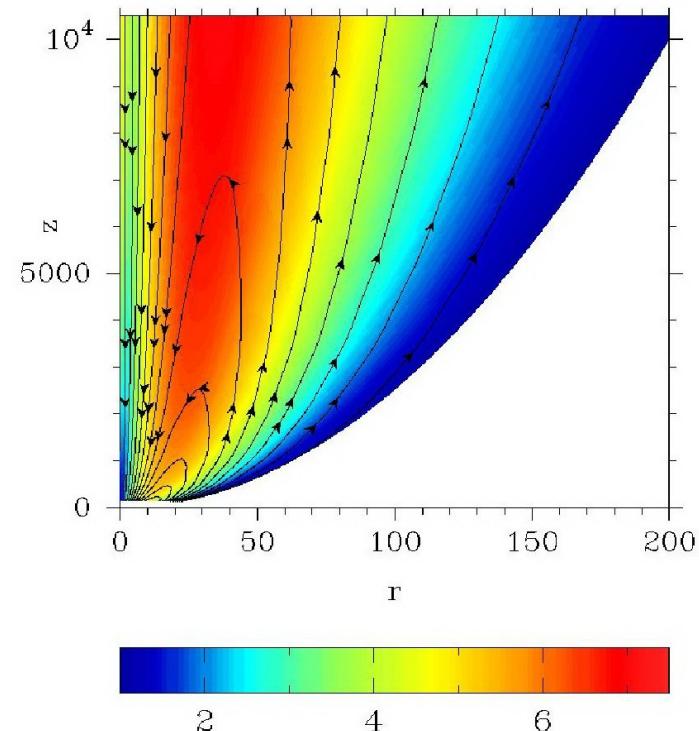


$T=30000$

$$\Omega = \Omega_0 \left[ 1 - 3 \left( \frac{\xi}{\xi_{\max}} \right)^2 + 2 \left( \frac{\xi}{\xi_{\max}} \right)^3 \right]$$

$$\mu = \Gamma(1 + \sigma) \approx 11$$

$\Gamma$

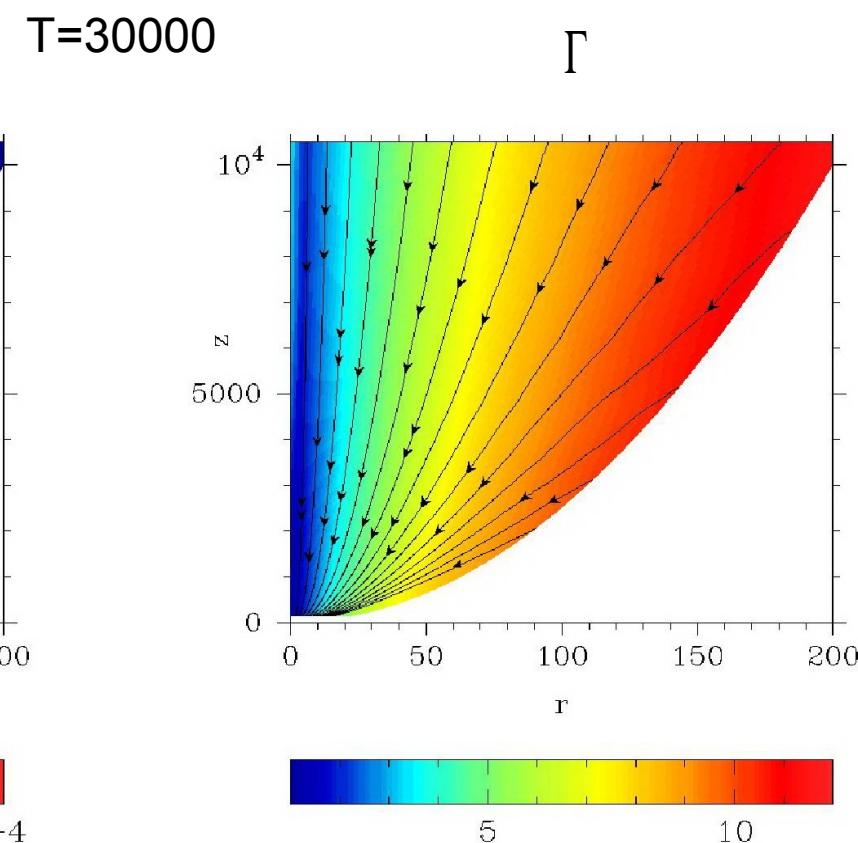
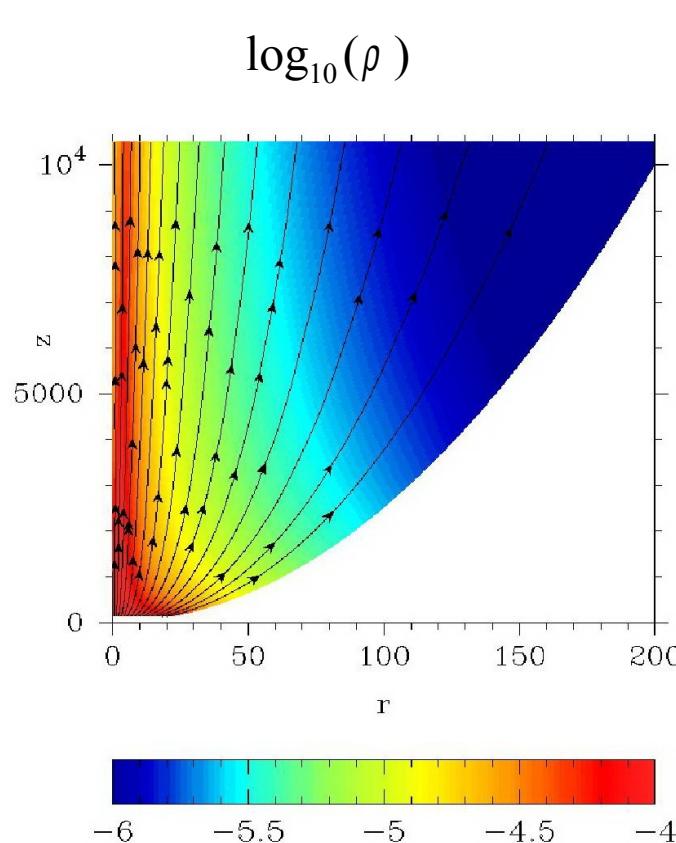


## Uniform rotation

$$z \propto r^2$$

$$\Omega = \Omega_0$$

$$\mu = \Gamma(1 + \sigma) \approx 16$$

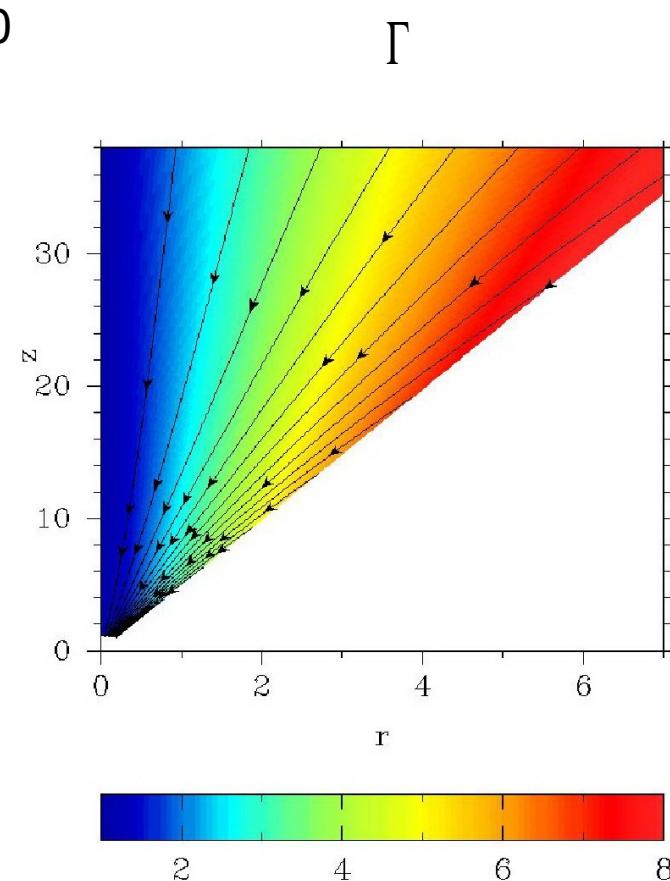
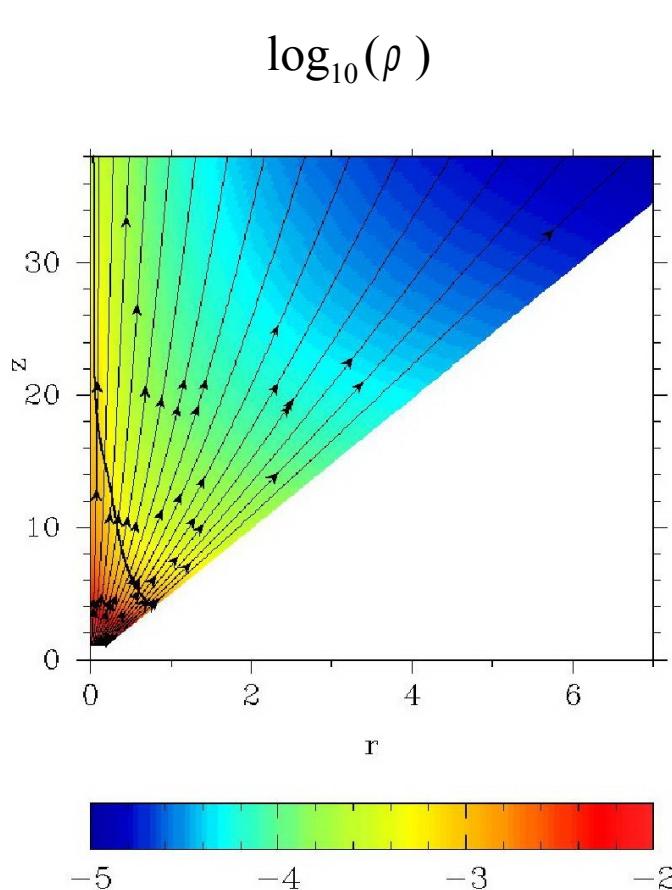


## Uniform rotation

$$z \propto r^1$$

$$\Omega = \Omega_0$$

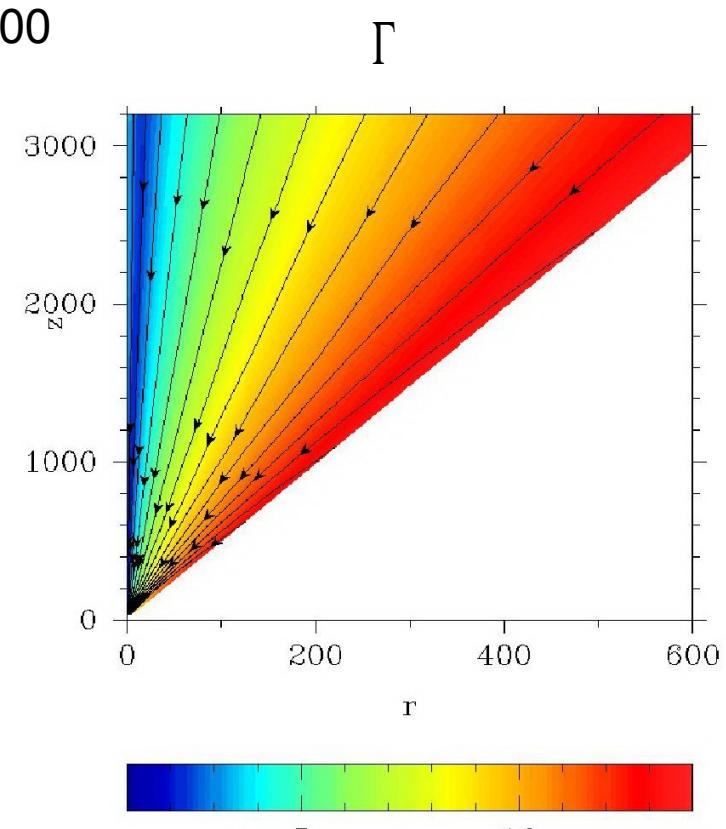
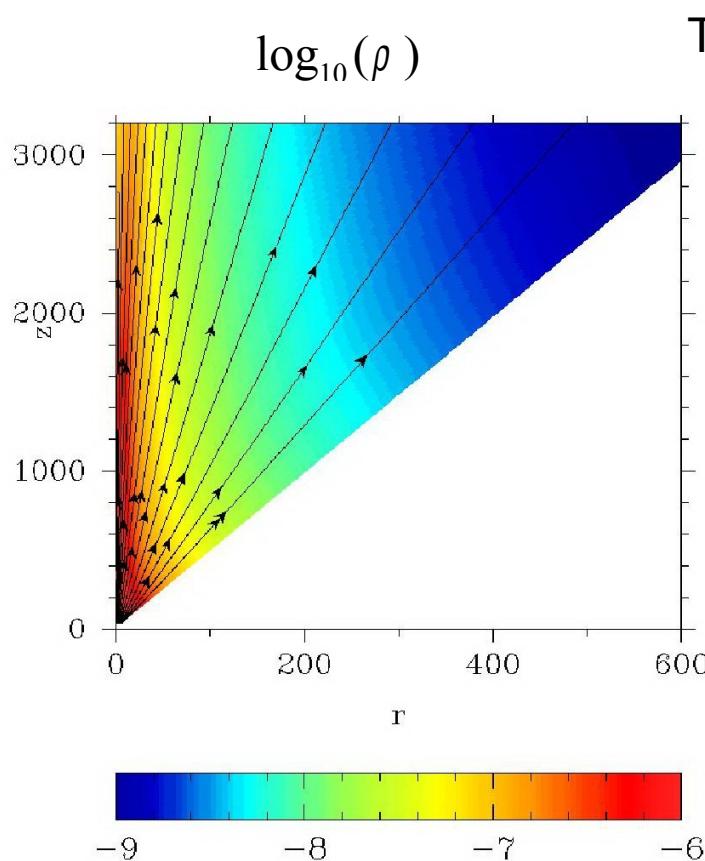
$$\mu = \Gamma(1 + \sigma) \approx 18$$



## Uniform rotation

$$z \propto r^1$$

$$\Omega = \Omega_0$$
$$\mu = \Gamma(1 + \sigma) \approx 18$$



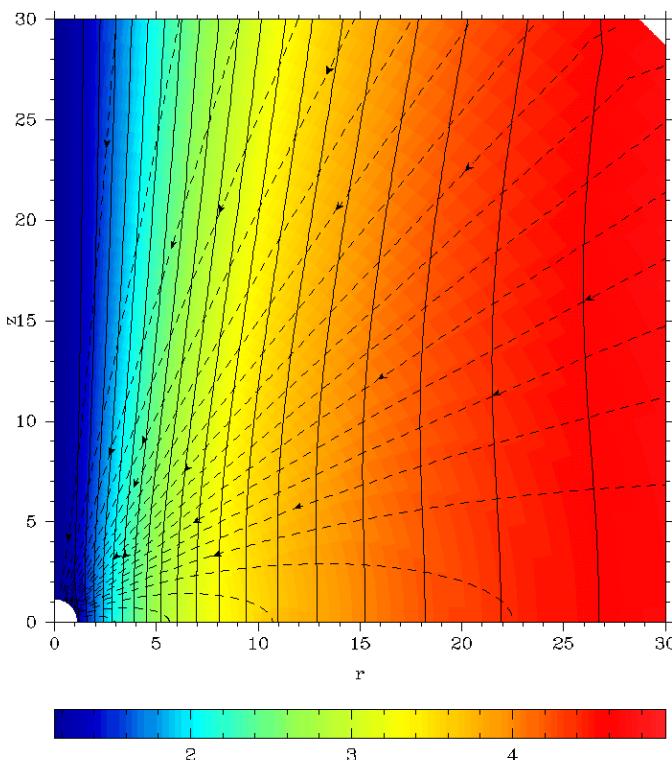
$$\Omega = \Omega_0$$

$$z \propto r^1$$

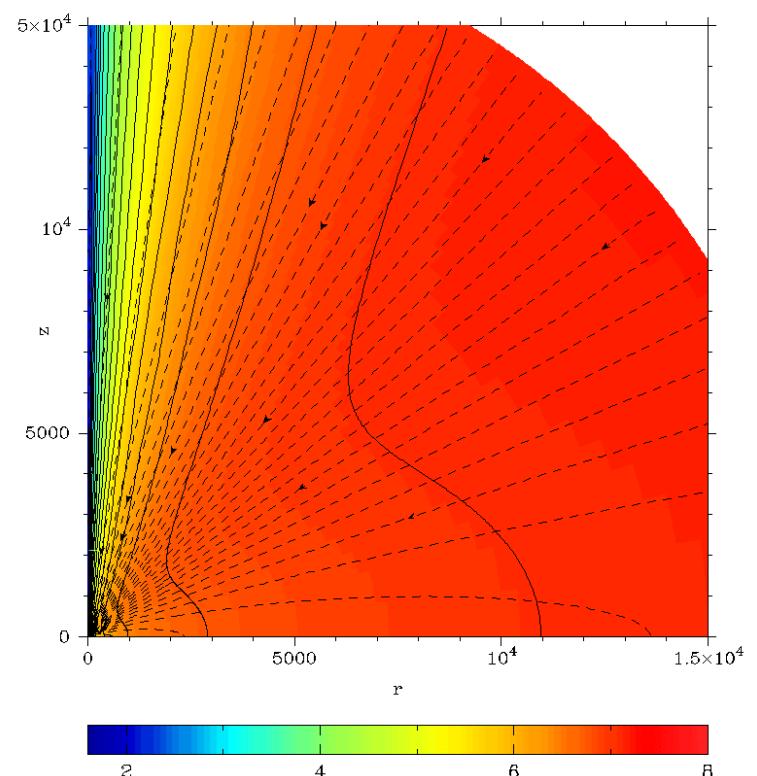
$$\mu = \Gamma(1 + \sigma) \approx 18$$

Lorenz factor distribution (color and solid lines) and current lines (dashed lines).

Central part of domain



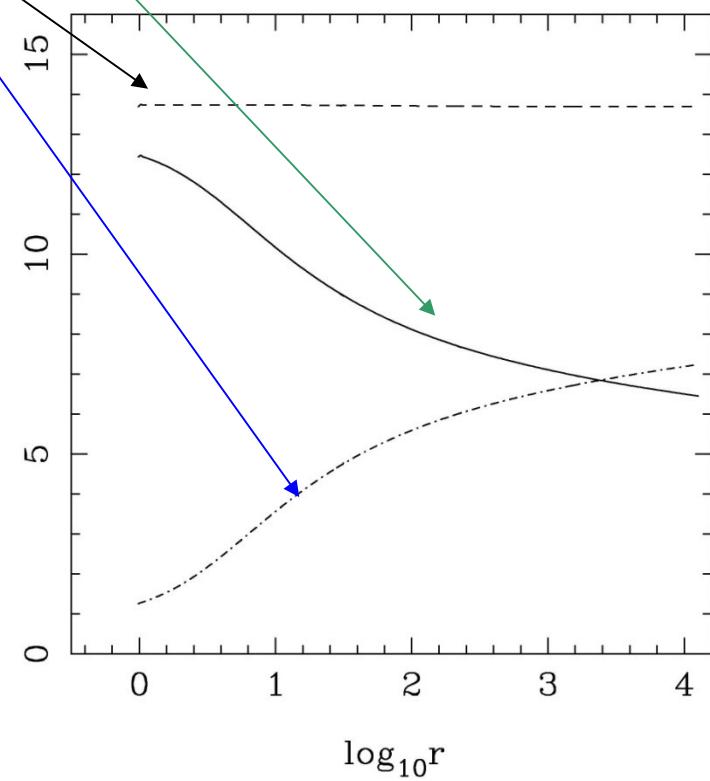
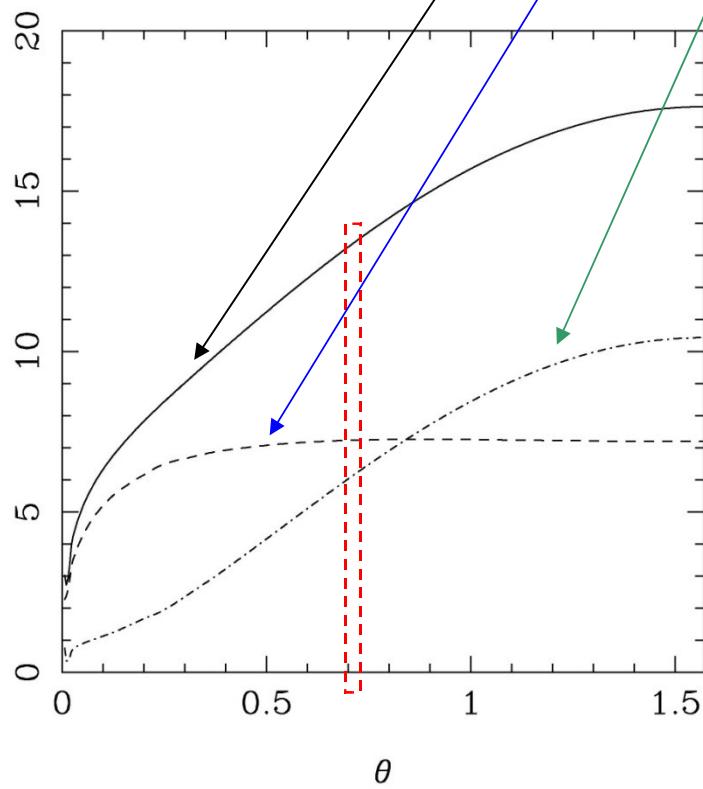
Full domain



$$\Omega = \Omega_0$$

$$\mu = \Gamma(1 + \sigma) \approx 18$$

$$z \propto r^1$$



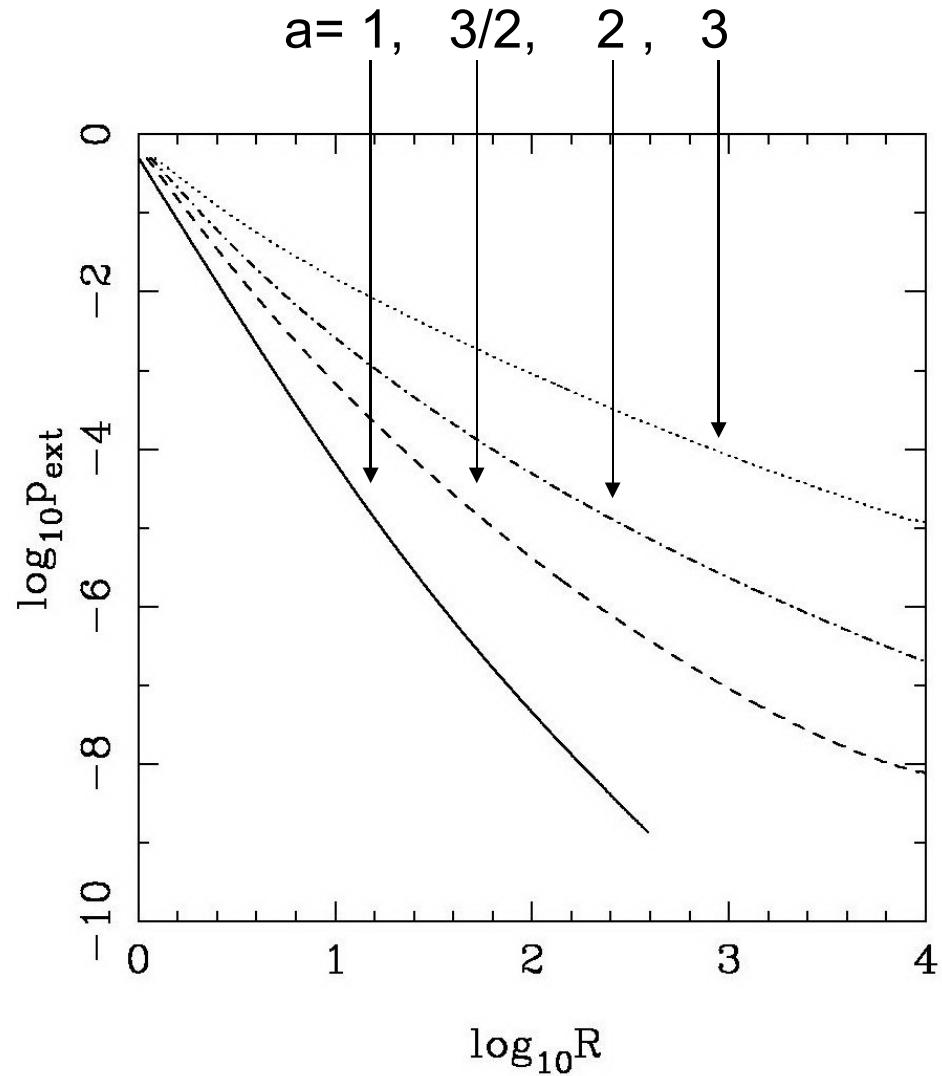
$$Z \propto r^a \quad \Omega = \Omega_0$$

The effective pressure deduction as a function of spherical radius.

We can calculate mean power-low indices  $p_{ext} = R^{-\alpha}$

a	1	3/2	2	3
$\alpha$	3.5	2	1.6	1.1

For example, in a spherical wind of polytropic index 5/3, the thermal pressure scales as  $R^{-10/3}$  and the ram pressure as  $R^{-5/2}$ . Thus, a disk wind that assumes a nearly spherical geometry not too far from the origin could effectively confine a relativistic jet with a nearly conical outer boundary.



## Application for AGN jets.

BH driven jet

$$r_{lc} \approx 4r_g = 6 \times 10^{13} \left( \frac{M}{10^8 M_{sun}} \right) cm$$

$$r_{eq} \approx 30r_{lc} \approx 2 \times 10^{15} \left( \frac{M}{10^8 M_{sun}} \right) cm$$

$$R_{eq} \approx 2 \times 10^{16} \left( \frac{M}{10^8 M_{sun}} \right) \left( \frac{0.1}{\Theta_j} \right) cm$$

Disk driven jet

$$r_{lc} \approx 5 \times 10^{14} \left( \frac{M}{10^8 M_{sun}} \right) \left( \frac{r_0}{10r_g} \right)^{3/2} \left[ \frac{\Omega_k(r_0)}{\Omega} \right] cm$$

$$r_{eq} \approx 30r_{lc} \approx 2 \times 10^{16} \left( \frac{r_{lc}}{5 \times 10^{14} cm} \right) cm$$

$$R_{eq} \approx 2 \times 10^{17} \left( \frac{r_{lc}}{5 \times 10^{14} cm} \right) \left( \frac{0.1}{\Theta_j} \right) cm$$

Where:

$$r_g \equiv \frac{GM}{c^2}$$

$r_{eq}$  - cylindrical  
 $R_{eq}$  - spherical

} radius of equipartition between Poynting and matter energy fluxes

$$\Omega = \Omega_0 \left[ 1 - 3 \left( \frac{\xi}{\xi_{\max}} \right)^2 + 2 \left( \frac{\xi}{\xi_{\max}} \right)^3 \right]$$

$$\mu = \Gamma(1 + \sigma) \approx 11$$

$$z \propto r^2$$

$$\Omega = \Omega_0$$

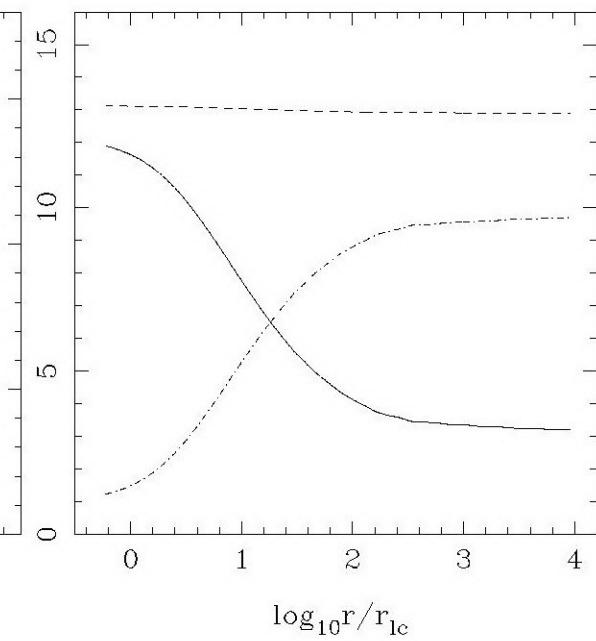
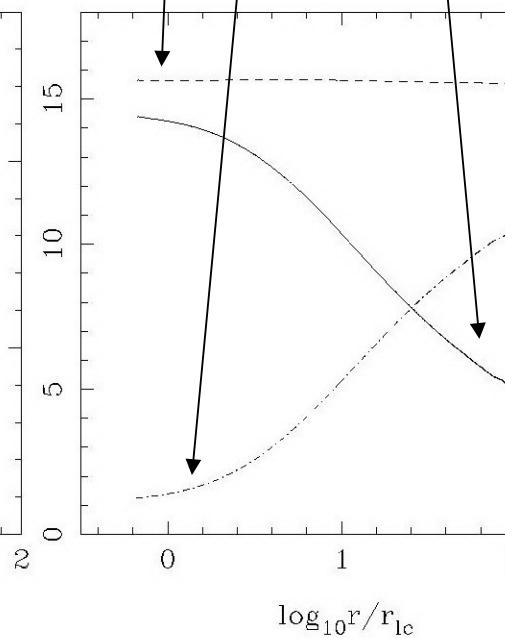
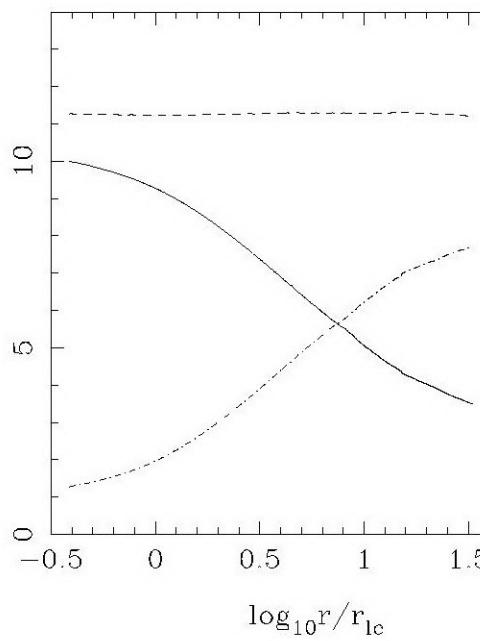
$$\mu = \Gamma(1 + \sigma) \approx 16$$

$$z \propto r^2$$

$$\Omega = \Omega_0$$

$$\mu = \Gamma(1 + \sigma) \approx 13$$

$$z \propto r^1$$



Our calculations could be implemented for GRB models as well.

The biggest Lorenz factor can reach value up to  $\Gamma = 150\text{-}200$ .

