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# Monte-Carlo code development for different astrophysical problems

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# General notes

- The Monte-Carlo method (s) is also called the statistical test method
- An innumerable number of processes can be listed the outcome of which is not determined and has probabilistic nature, but these processes obey statistical laws, so to obtain the result we have to make multiple similar tests, i.e. to use the MC method
- Because probabilistic processes are widely spread in nature, the MC method is used in many fields of science from economics to astrophysics and particle physics
- It is possible to apply the MC method to transfer problems by simulating the particle’s motion and its interaction with the medium
- From the Monte-Carlo method point of view there is little difference between different sorts of particles, so it can easily applied to photon, neutron or neutrino transfer

# The advantages of the Monte-Carlo method

- the computational algorithm is rather simple, it roughly consists of a single procedure repeated multiple times (a great number)

- the method allows us to account for numerous interaction processes that is impossible analytically

- the method is very efficient when used on parallel computers, no accumulation of errors

# The structure of a single test

1. Initial data input:

- coordinates (place of birth or coordinates on the boundary of the computational domain)
- direction of motion (momentum vector)
- energy of the particle
- other particle's characteristics (polarization, spin, flavor)

## 2. Trajectory simulation: what we should do

- infer of flight path length before an act of interaction occurs

- choose the interaction process

- infer the particle’s state after interaction (energy, direction of motion, etc.)

Repeat this procedure until our particle escapes the computational domain or is absorbed in some process.

## 3. Information gathering

After the particle escapes the domain we should gather the information about its final state.

# Important particles

There are two kinds of particles that play the most important role in astrophysics: photons and neutrino.

Photons:

- photons from astrophysical sources allow us to study their structure
- most observational instruments are made for photon detection

Neutrino:

- a very important role in SN type II explosions, most energy is carried by neutrino
- neutrino background arising from SN explosions

# The model particles problem

- It is obvious that one cannot simulate the trajectories of all photons emitted from the source, so we should introduce model photons and their number should be arbitrary. As a consequence of the method's statistical nature the relative error of the result will decrease proportional to square root the number of tests.

- It was suggested by L.B. Lucy that we should group photons into packets of **constant energy**, that will be model photons. Such trick helps us to avoid simulating trajectories of a great number of low-energy photons.

$$\varepsilon_0 = nh\nu$$

- It is convenient to assume that model photons interact with medium **as a whole**, the process of interaction should be chosen randomly from a set of possible (or important for particular spectral range), so the splitting of photon packets and the following branching of computational algorithm is avoided.

Lucy L.B., 1999, Astron.Astrophys., 344, 282

# The number of photons and their spectral distribution

A spectral range of our interest should be chosen and somehow divided into parts or frequency bins, denoted by boundary frequencies $\nu_0, \nu_1, \nu_2, ..., \nu_k$.

The total number of photons participating in the Monte-Carlo experiment $N$ is arbitrary. It determines the precision of the result.

With the total luminosity of the source known, we obtain the value

$$\frac{\varepsilon_0}{\Delta t} = \frac{L_{tot}}{N}$$

$\varepsilon_0$ - the energy of model photon (photon packet)

$\Delta t$ - is the duration of the MC experiment (the greatest photon escape time)

In case of several spectral components contributing to the total luminosity, the number of photons representing each component is

$$N_i = \frac{L_i}{\varepsilon_0 / \Delta t} = \frac{L_i}{L_{tot}} N$$

So, the number of photons of i-th spectral component in j-th frequency bin is

$$N_{ij}/N_i = \int_{\nu_{j-1}}^{\nu_j} L_{i,\nu}\, d\nu / \int_0^{\infty} L_{i,\nu}\, d\nu$$

We simulate photon's trajectory as in propagates through the medium. Escaped photon packets make their contribution to the source's spectrum. The algorithm for simulating the photon's trajectory is the following:

1. Determination of optical depth a packet must pass to interact with the medium according to formula $\tau = -\ln\gamma, \quad \gamma$ is a random number.

2. Determination of coordinates of the interaction point

$$\tau = \int dl n_e(r)\left[\sigma_C + k_\nu(T) n_e(r)\right]$$

trajectory – straight line

$$\vec{r}_{i+1} = \vec{r}_i + s\vec{e}_v$$

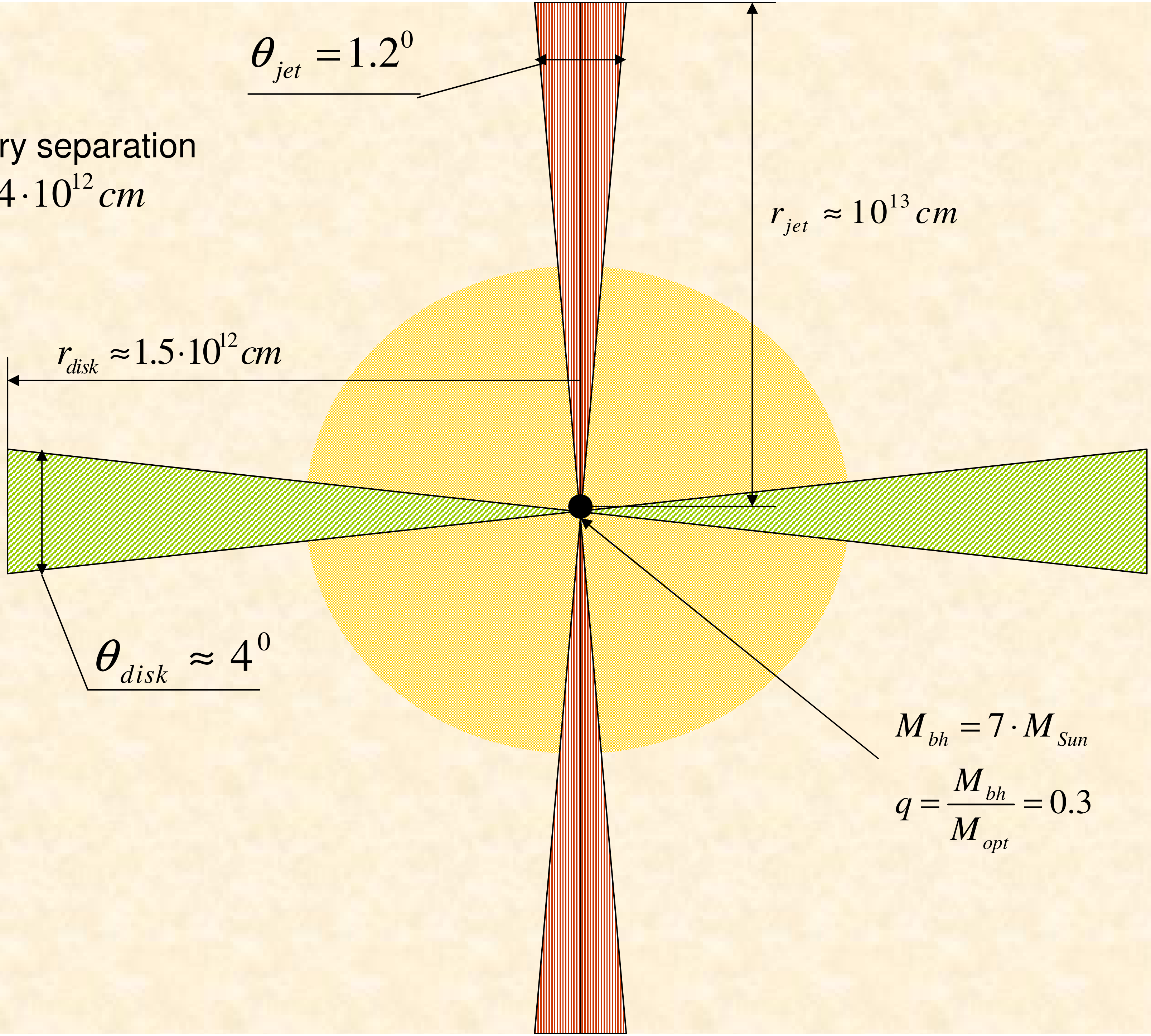
3. Choosing event (Compton scattering or free-free absorption), according to the criterion: $\gamma < \frac{\sigma_C}{\sigma_C + k_\nu(T) n_e(r)}$

< - Compton scattering

> - free-free absorption

4. Determination of photon's new frequency and direction of motion in case it suffered scattering

These steps should be repeated consequently until photon gets absorbed or escapes from the computational domain.



$\theta_{jet} = 1.2^0$

Binary separation

$a = 4 \cdot 10^{12}\, cm$

$r_{jet} \approx 10^{13}\, cm$

$r_{disk} \approx 1.5 \cdot 10^{12}\, cm$

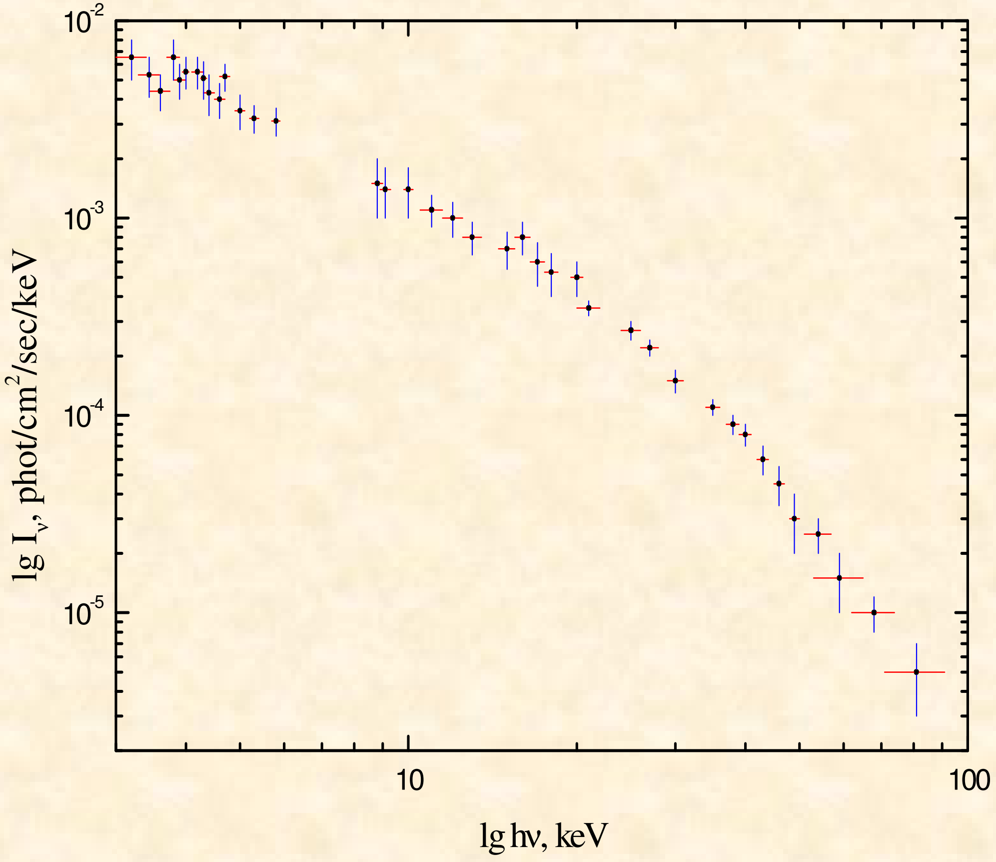
$\theta_{disk} \approx 4^0$

$M_{bh} = 7 \cdot M_{Sun}$

$q = \frac{M_{bh}}{M_{opt}} = 0.3$

# Observational data

In this figure the SS433 spectrum in the range from 3 to 90 keV is presented. It was obtained from INTEGRAL data (JEM-X points from 3 to 20 keV and IBIS (ISGRI) points from 20 to 90 keV). The spectum corresponds to precessional moment T3, i.e. when the angle between jet axis and the line of sight is equal 60 degrees and the disk is maximally 'face-on'.



Cherepashchuk A.M.,Sunyaev R.A., Fabrika S.N., Postnov K.A. et al., 2005, A&A, 437, 561

Cherepashchuk A.M.,Sunyaev R.A. et al., 2006, Proceeding of 6th INTEGRAL Workshop, Moscow, Russia

# The jet

It follows from observations, that jet's opening angle in X-ray and optical range is the same and is equal 1.2 degrees. That leads us to the assumption that jet is of conical shape.

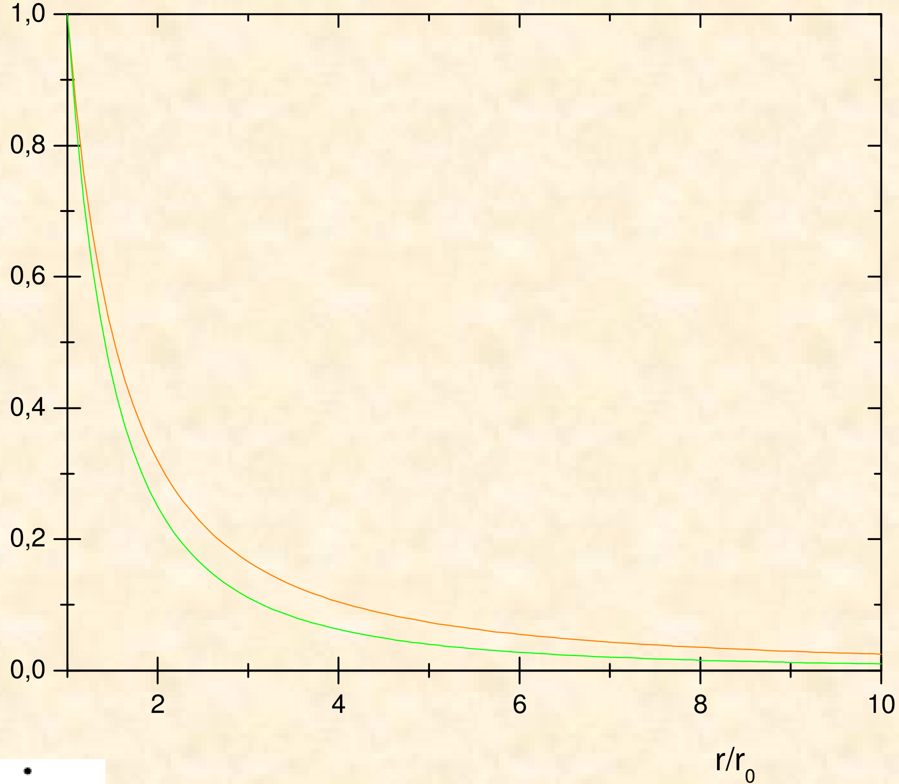
Temperature profile: $T = T_0 \left(\frac{r_0}{r}\right)^{4/3}$

This formula corresponds to adiabatic cooling of expanding ideal gas.

Density profile: $n = n_0 \left(\frac{r_0}{r}\right)^2$

This formula can be obtained from the equation of continuity with the following expression for n0:

$$n_0 = \frac{\dot{M}_{jet}}{\pi m_p V r_0^2} \tan^2 \frac{\theta_{jet}}{2}$$



$\dot{M}_{jet}$ is mass loss rate in the jet

$V = 0.26c$ is the radial velocity in the jet

# The corona

- The corona has a spherical shape, its inner radius is $r_0$ , the outer one is $r_{cor}$ .

- It was considered to be isothermal, with temperature equal to $19keV = 2.2 \cdot 10^8 \text{ K}$

- The density profile was taken to be the same as in the jet for simplicity, but with different value at $r_0$.

$$\tau_{cor} = \sigma_T n_0 \int\limits_{r_0}^{r_{cor}} \frac{r_0^2}{r^2}\, dr$$

- optical depth of corona with respect to Thomson scattering

And thus we can obtain the formula for the outer radius of the corona:

$$r_{cor} = r_0 / \left(1 - \frac{\tau_{cor}}{\sigma_T n_{cor} r_0}\right)$$

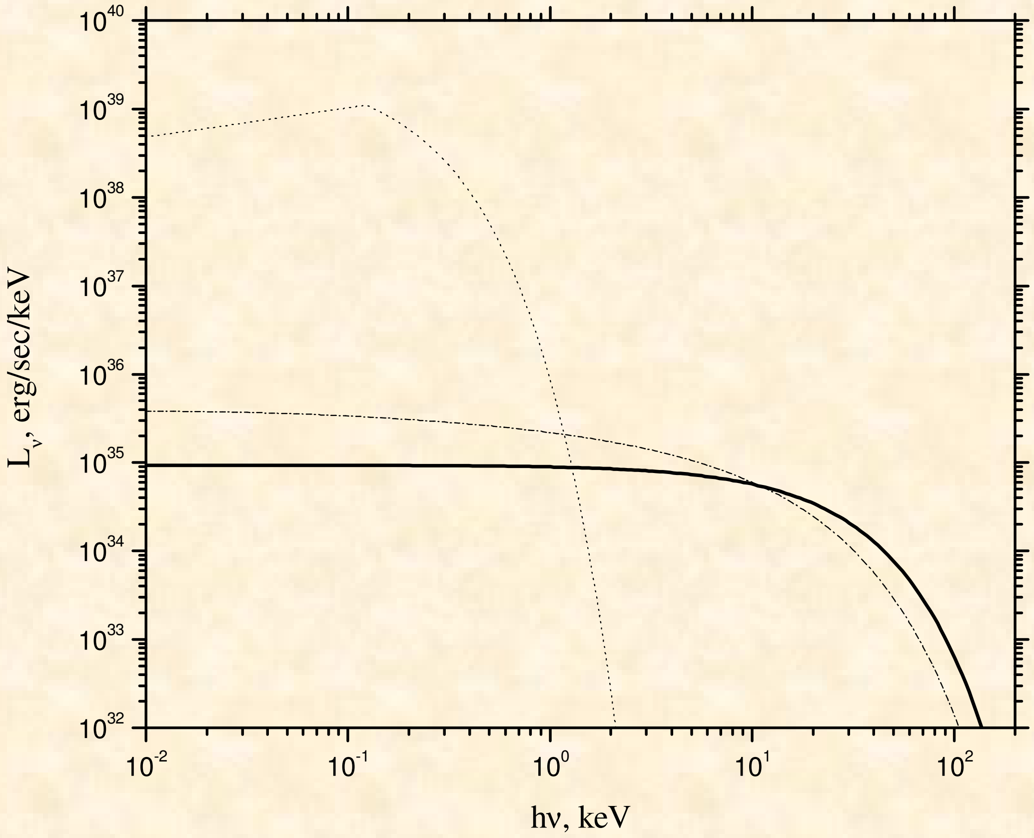
# Initial spectra

Initial spectra are presented:

-**bold solid line** corresponds to corona free-free emission with 19 keV temperature

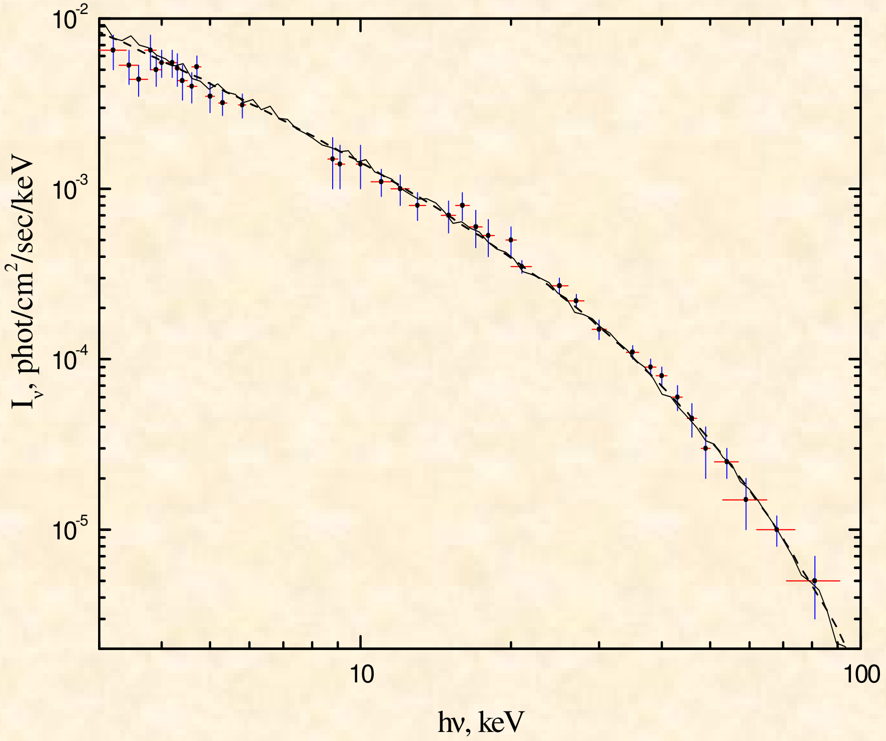
- dash-dotted line – jet free-free spectrum

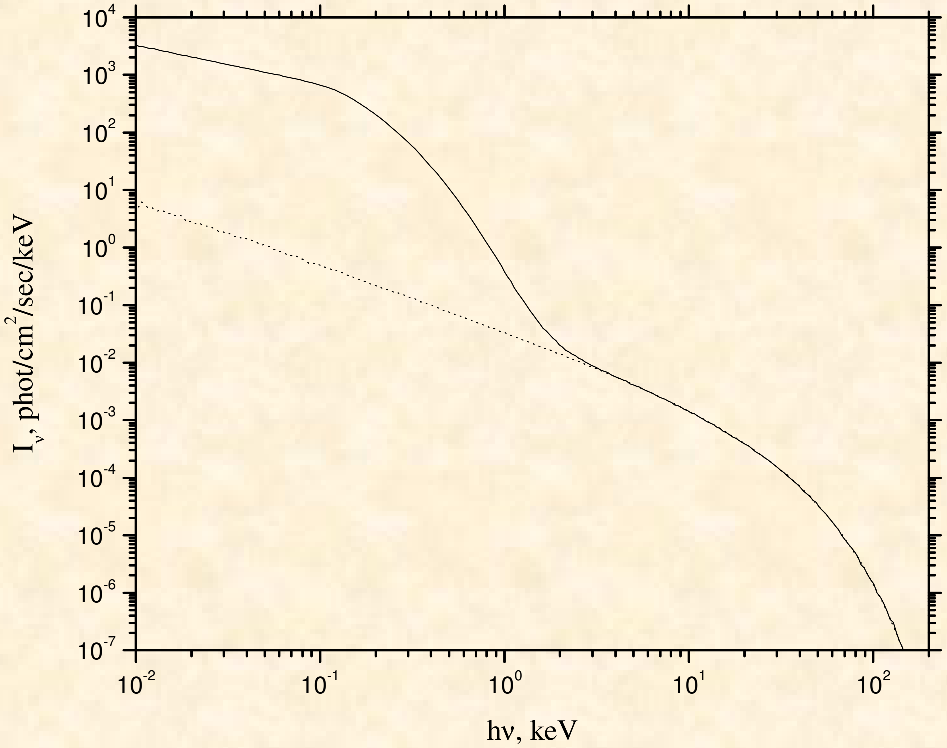
-dotted line – accretion disk radiation spectrum



**The resulting X-ray spectrum is formed by these three components due to Comptonization of photons in hot corona**

# The results of simulations





*Comparison of results with observational data obtained by INTEGRAL*

In the range of observations. Good agreement with the experiment.

In wide spectral range. Solid curve corresponds to the model that includes disk radiation, dashed one - to the model without disk radiation.

**Yu.M. Krivosheyev, G.S. Bisnovatyi-Kogan, A.M. Cherepaschuk, K.A. Postnov, 2008(9), MNRAS submitted**

# What about neutrino?

The simulation scheme is the same for photon and neutrino transfer.

Differences:

- they obey different statistics (Pauli exclusion principle)
- different processes of creation
- different processes of interaction with medium

What processes are of creation and interaction are important?

**Neutrino creation processes:**

- -Urca-processes (emission in hadronic processes)
- - photon-electron scattering creation
- -electron-positron annihilation
- - plasmon decay
- - other minor processes (photon-photon interaction, photo-nuclear neutrino creation, etc.)

**Interaction processes:**

- - absorption in hadronic processes (electrons and positrons created)
- - neutrino scattering off nucleons
- -neutrino scattering off leptons
- - …

Numerous creation/interaction processes just as in the photon case

# Conclusion

- the Monte-Carlo method is an essential choice when dealing with transfer problems
- the application of the MC method to photon transfer can provide enough accurate results for reasonable computational time that can be compared with the experiment
- the MC simulation can be incorporated into  hydrodynamic code to provide accurate treatment of energy gains and losses in self-consistent problems (SN explosion simulation)