Гравитационное линзирование при наличии плазмы и сильных гравитационных полей

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Структура доклада

1. Слабое гравитационное линзирование в вакууме
2. Сильное гравитационное линзирование на Шварцшильдовской черной дыре
3. Гравитационное линзирование в плазме, *гравитационный радиоспектрометр*

Einstein’s deflection law

vacuum

General Relativity predicts that a light ray which passes by a spherical body of mass M at a minimum distance ξ, is deflected by the "Einstein angle":

$$\hat{\alpha} = \frac{4GM}{c^2\xi} = \frac{2R_S}{\xi},$$

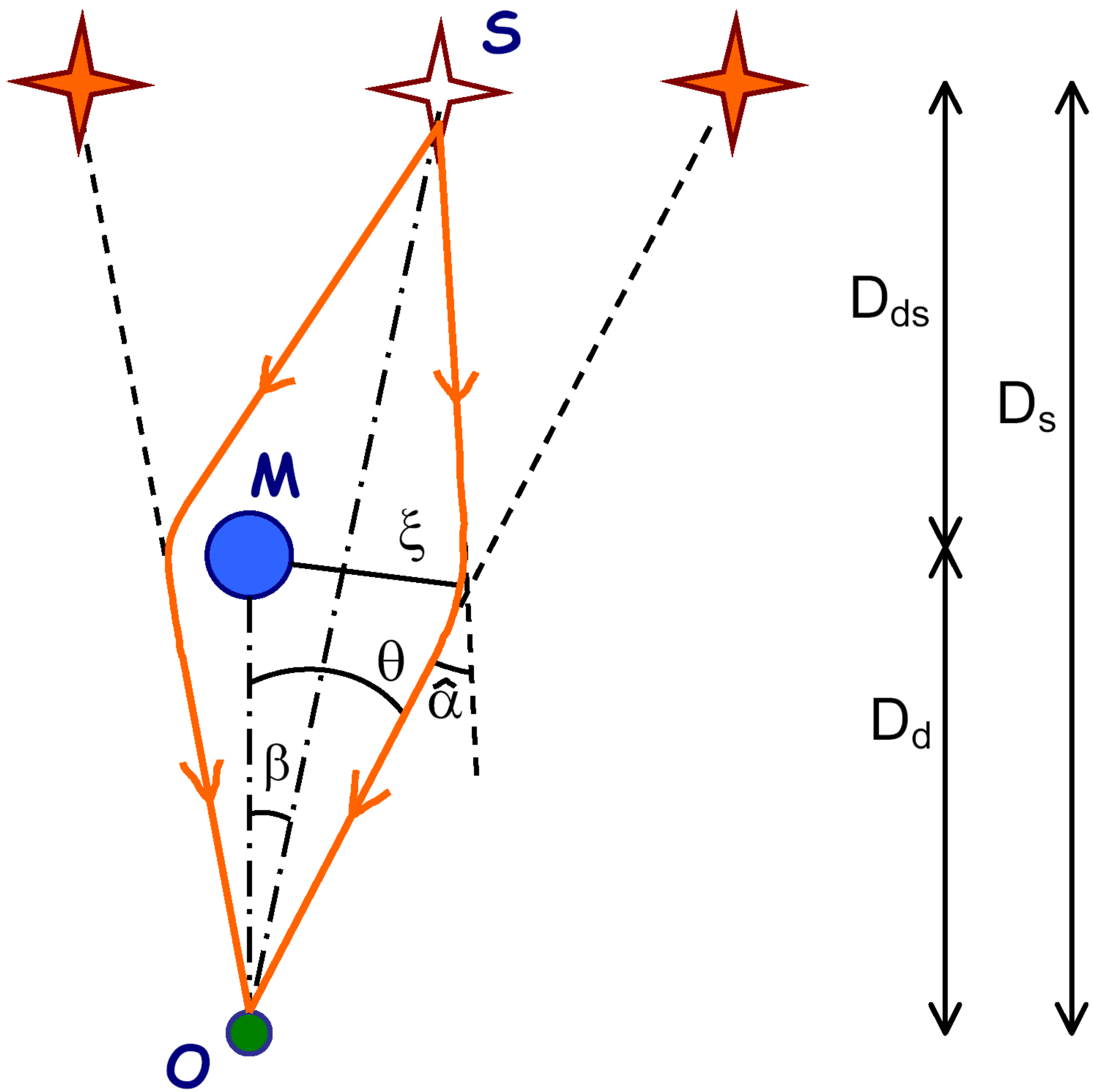
provided the impact parameter ξ is much larger than the corresponding Schwarzschild radius R_S:

$$\xi \gg R_S = \frac{2GM}{c^2}.$$

In the most astrophysical situations related with gravitational lensing approximation of weak deflection is well satisfied.

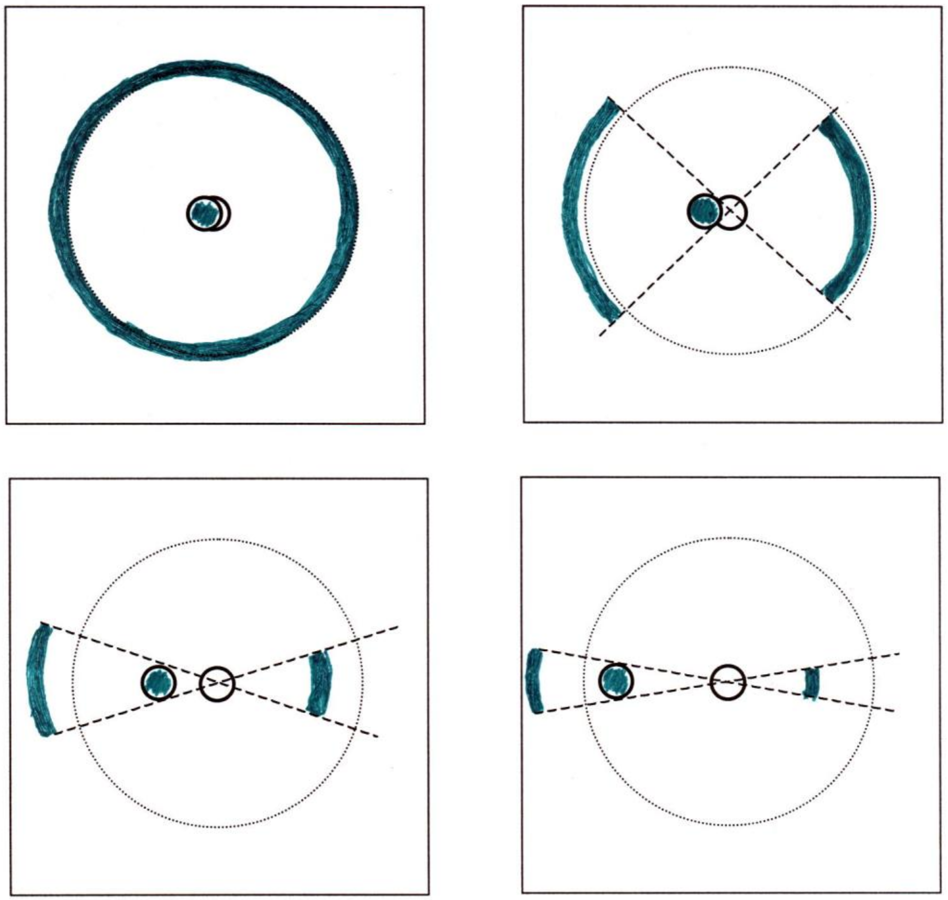
This angle does not depend on frequency of the photon

The simplest model of the Schwarzschild point-mass lens



On basis of Einstein deflection angle ordinary GL theory is developed. At this picture there is the example of the simplest model of Schwarzschild point-mass lens which gives two images of source instead of one single real source.

Arcs (дуги)



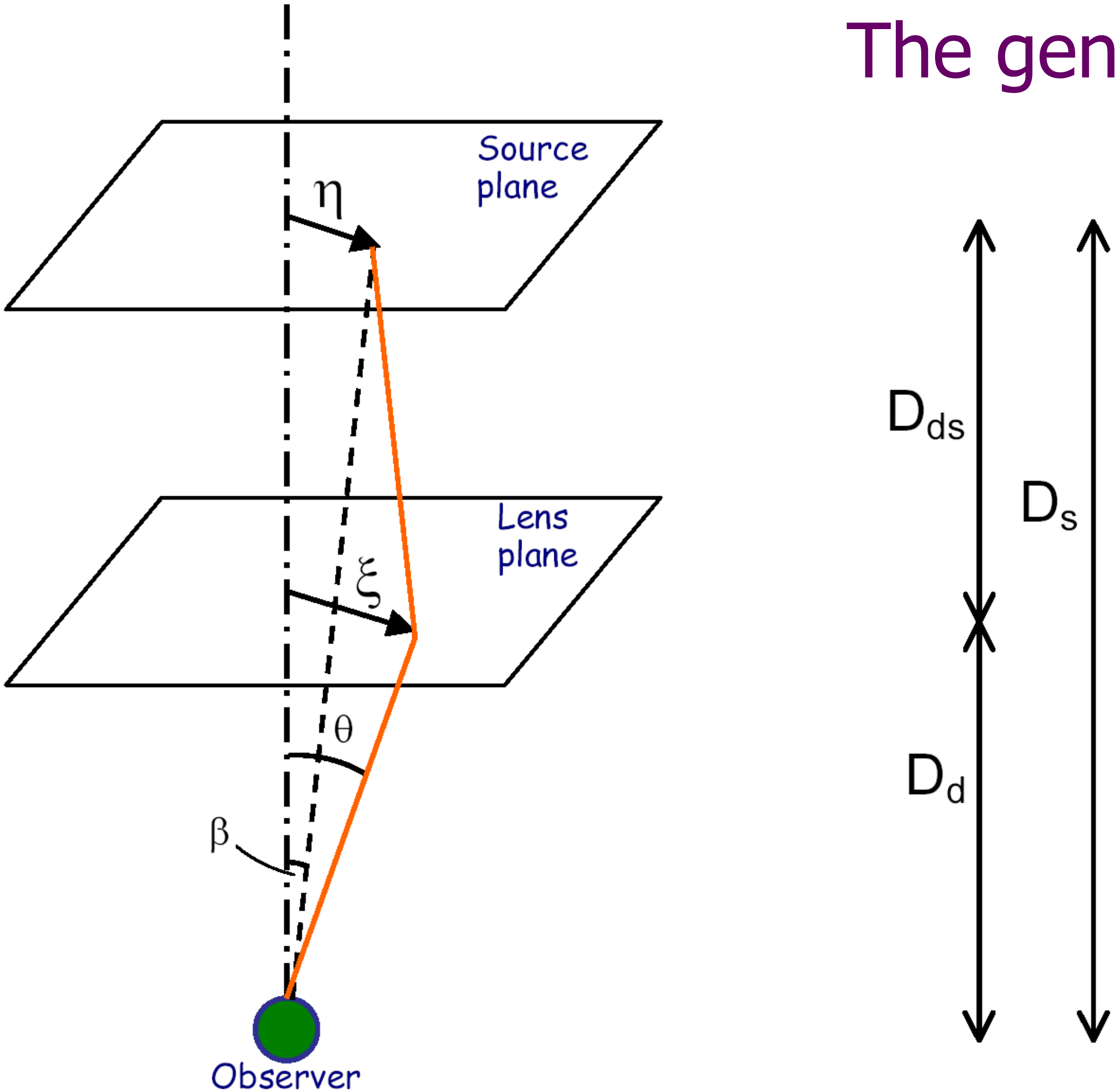
The magnification factor

The surface brightness I for an image is identical to that of the source in the absence of the lens. The flux of an image of an infinitesimal source is the product of its surface brightness and the solid angle $\Delta\omega$ it subtends on the sky.

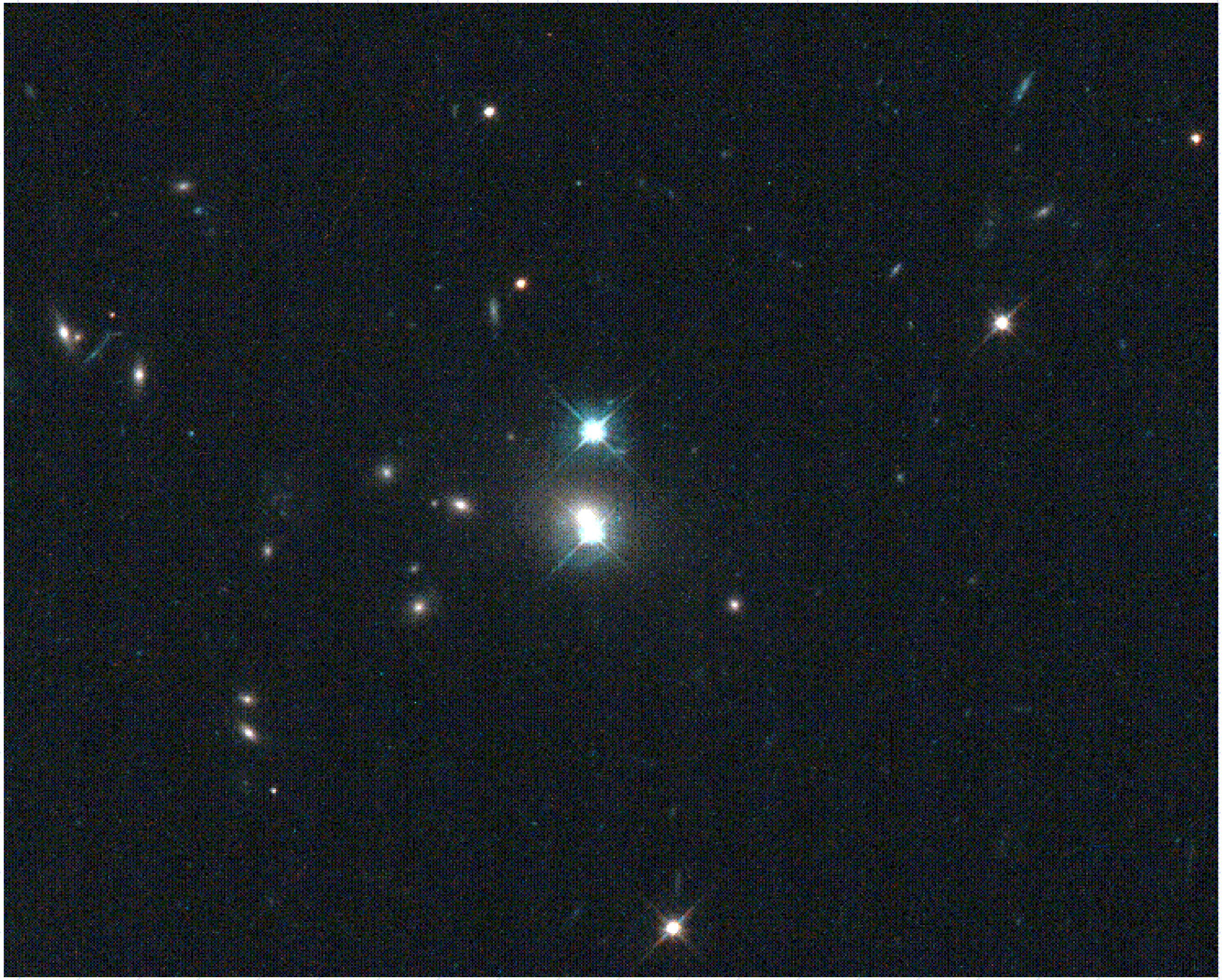
The magnification μ is the ratio of the flux of an image to the flux of the unlensed source:

$$\mu = \frac{\Delta\omega}{(\Delta\omega)_0}$$

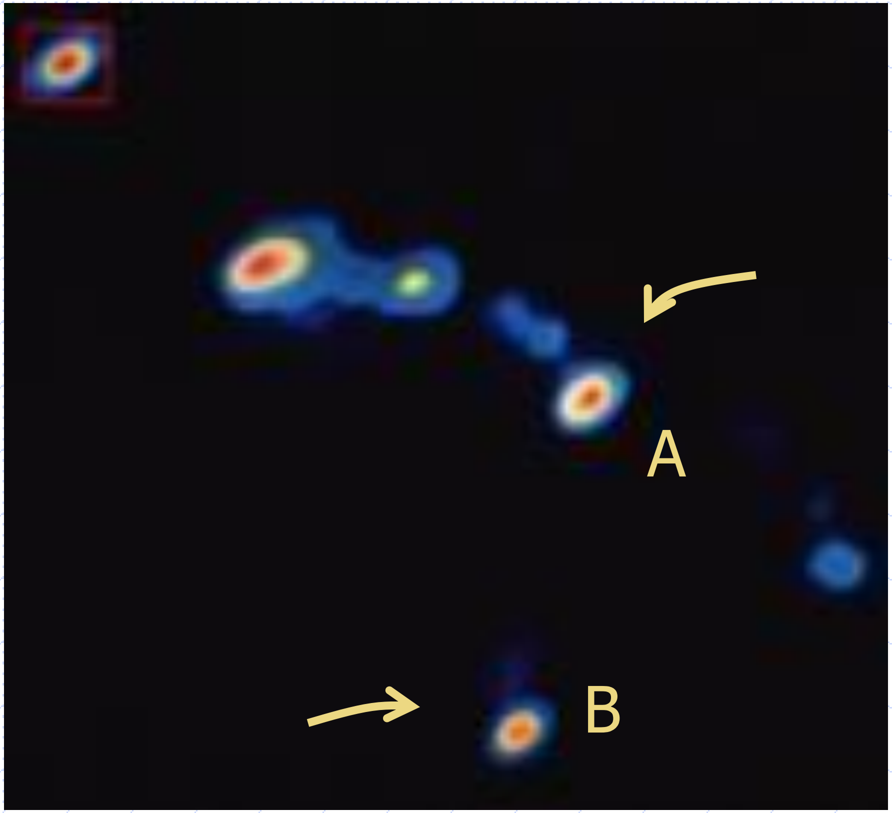
The general lens



The first observed lense (1979): QSO 0957+561, optics



QSO 0957+561, radio



- Maximum separation – 6.1 arcsec
- Image redshift – 1.41
- Lens redshift – 0.36
- B/A = 2/3

B1938+666

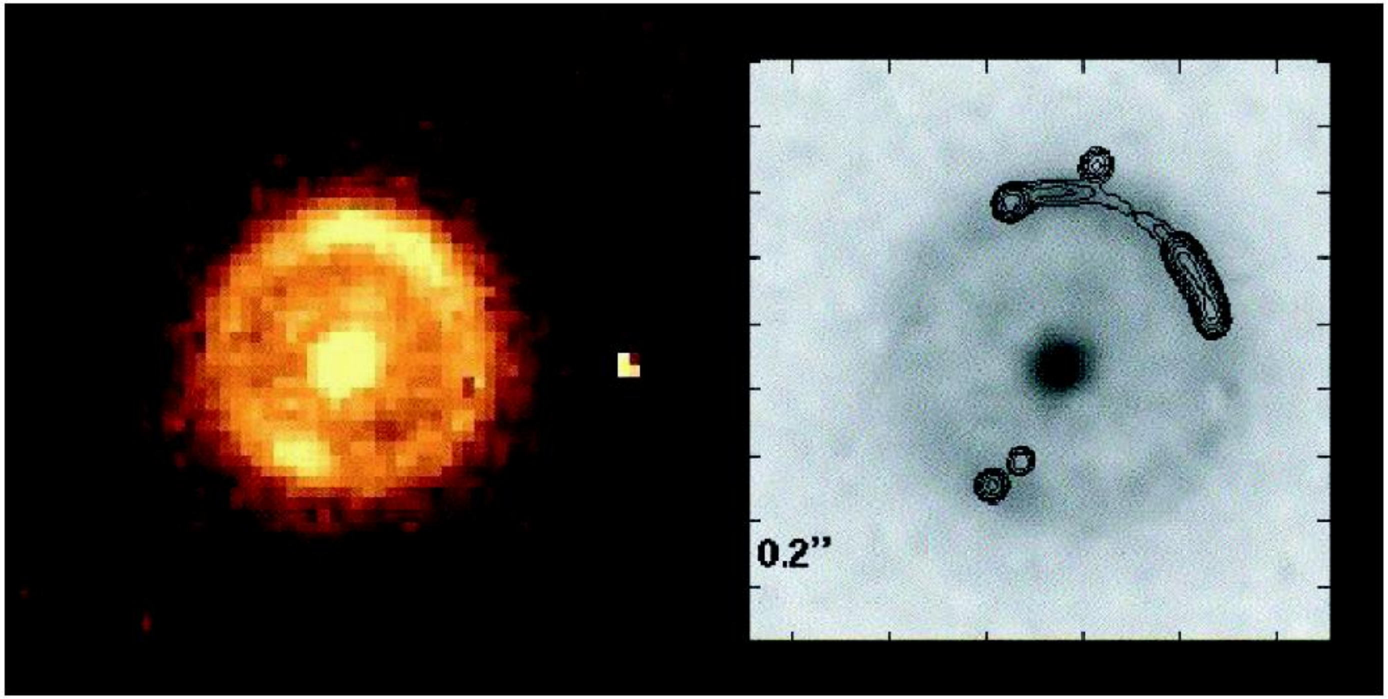
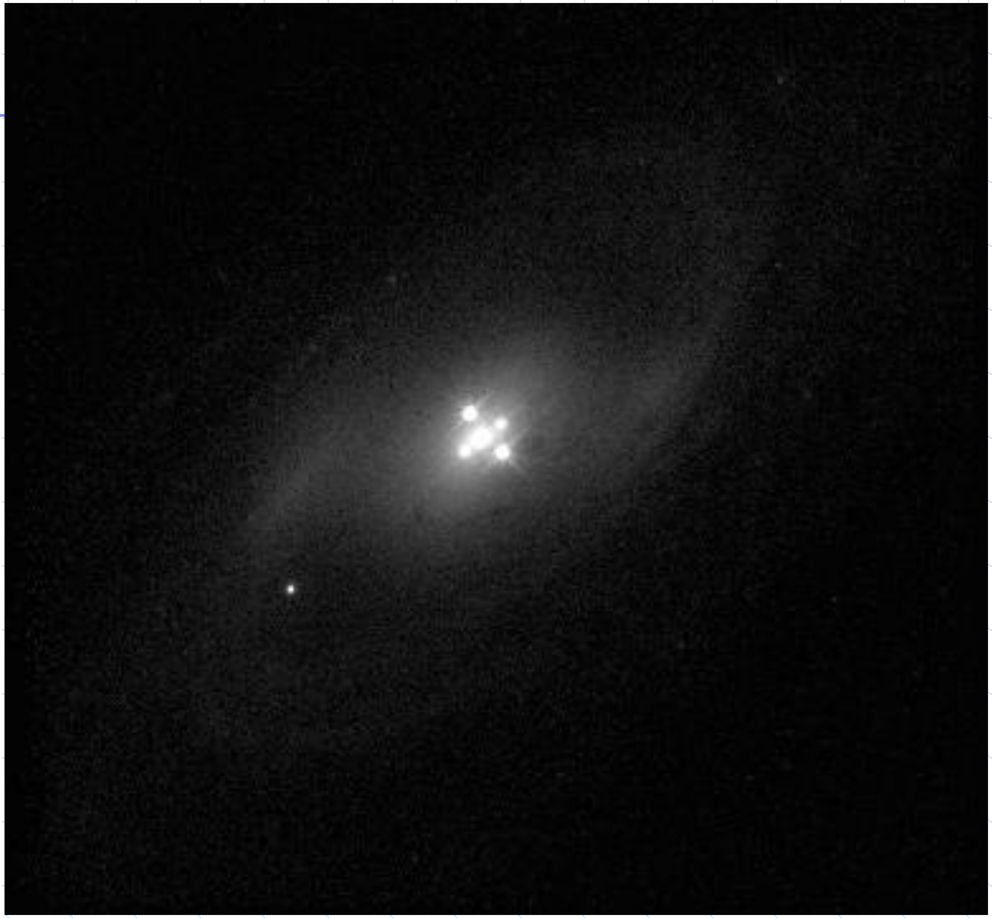


Fig. 8. The gravitational lens system B 1938+666. The *left panel* shows a NICMOS@HST image of the system, clearly showing a complete Einstein ring into which the Active Galaxy is mapped, together with the lens galaxy situated near the center of the ring. The *right panel* shows the NICMOS image as gray-scales, with the radio observations superposed as contours. The radio source is indeed a double, with one component being imaged twice (the two images just outside and just inside the Einstein ring), whereas the other source component has four images along the Einstein ring, with two of them close together (source: L.J. King, see King et al. 1998)

The Einstein Cross

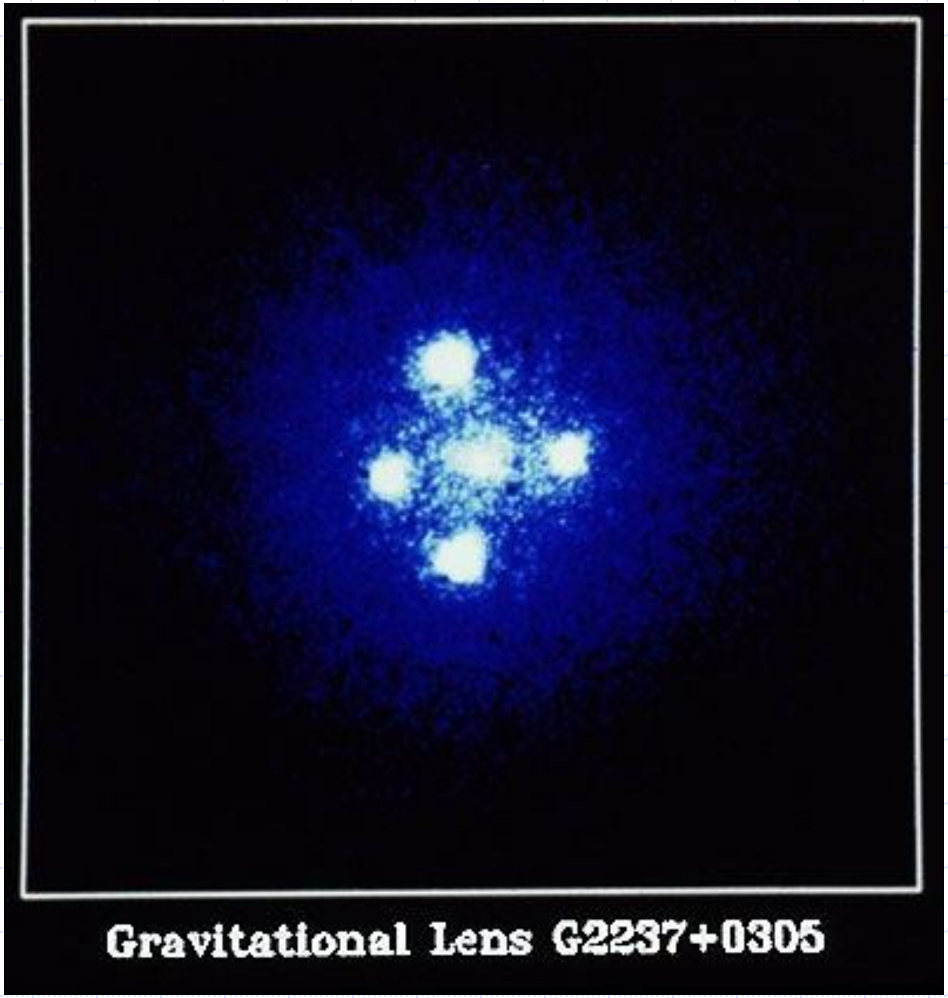


- Q2237+0305
- 1985, Huchra and others
- It is observed: four QSO images arrayed around the nucleus of the galaxy.
- The model : 1988, Schneider and others

This picture of the gravitationally lensed quasar Q2237+0305 and the associated lensing spiral galaxy was taken by the 3.5-meter WIYN telescope, on the night of October 4, 1999.

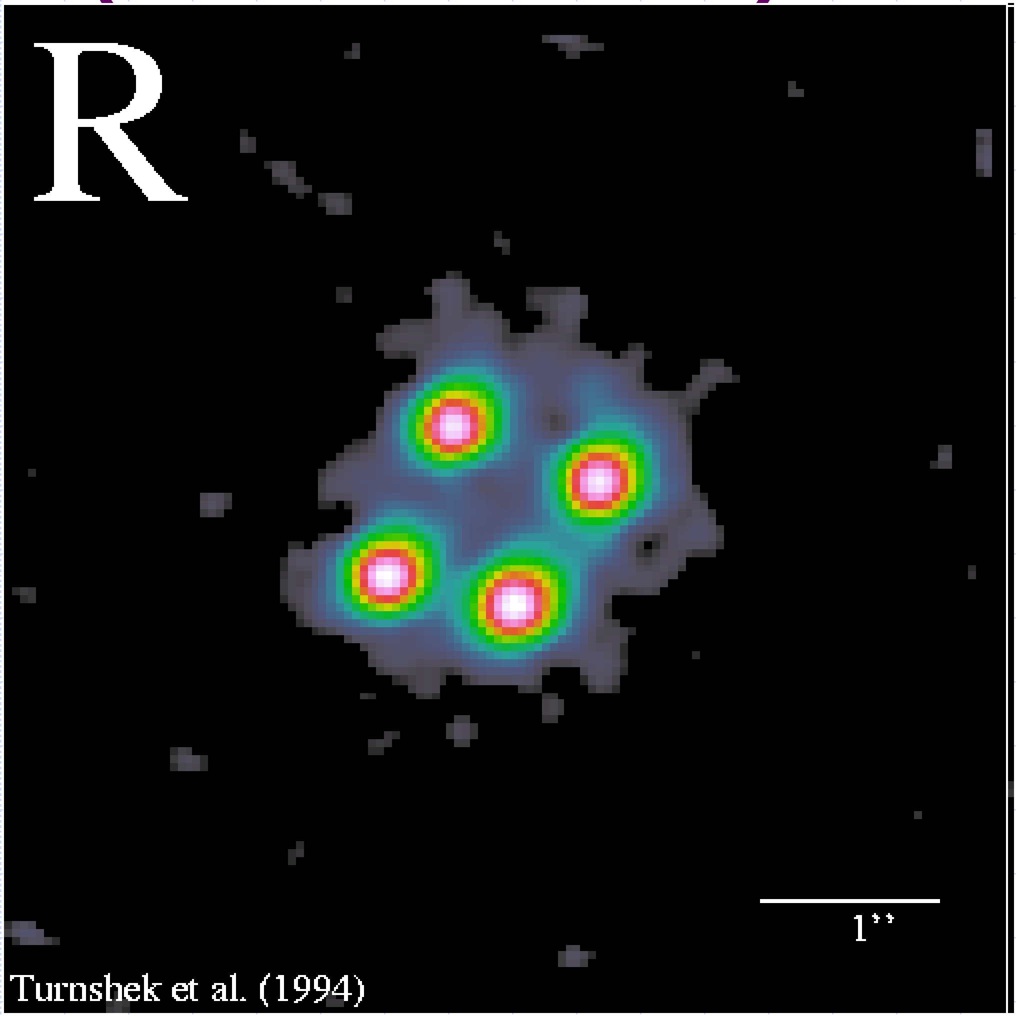
http://antwrp.gsfc.nasa.gov/apod/ap070311.html

The Einstein Cross



- The European Space Agency's Faint Object Camera on board NASA's Hubble Space Telescope has provided astronomers with the most detailed image ever taken of the gravitational lens G2237 + 0305—sometimes referred to as the "Einstein Cross". The photograph shows four images of a very distant quasar which has been multiple-imaged by a relatively nearby galaxy acting as a gravitational lens. The angular separation between the upper and lower images is 1.6 arc seconds.

Cloverleaf Quasar (H1413+117)



- Hubble Optical Image of Cloverleaf Quasar

http://chandra.harvard.edu/photo/2004/h1413/more.html

2. Strong gravitational lensing by Schwarzschild black hole

G.S. Bisnovatyi-Kogan, O.Yu. Tsupko

Strong gravitational lensing by Schwarzschild black holes, Astrofizika, Vol. 51, No. 1, pp. 125–138 (February 2008).

Let us consider the motion of a photon in the neighborhood of a black hole with a Schwarzschild metric. We shall work with a system of units in which the Schwarzschild radius $R_S = 2M$ $(G=1,\ c=1)$, where M is the mass of the black hole. The Schwarzschild metric is given by

$$ds^2 = g_{ik}\,dx^i\,dx^k = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

- Equations for the photon:

$$\left(\frac{dr}{d\lambda}\right)^2 + B^{-2}(r) = b^{-2}\ ,$$

$$\frac{d\varphi}{d\lambda} = \frac{1}{r^2},$$

$$\frac{dt}{d\lambda} = b^{-1}\left(1 - \frac{2M}{r}\right)^{-1}.$$

Here $B^{-2}(r) = \left(1/r^2\right)\left(1 - (2M/r)\right)$ is the effective potential and b is the impact parameter.

1. If $b < 3\sqrt{3}\,M$, then the photon falls to $R_S = 2M$ and is absorbed by the black hole.

2. If $b > 3\sqrt{3}\,M$, then the photon is deflected by an angle $\hat{\alpha}$ and flies off to infinity. Here there are two possibilities:

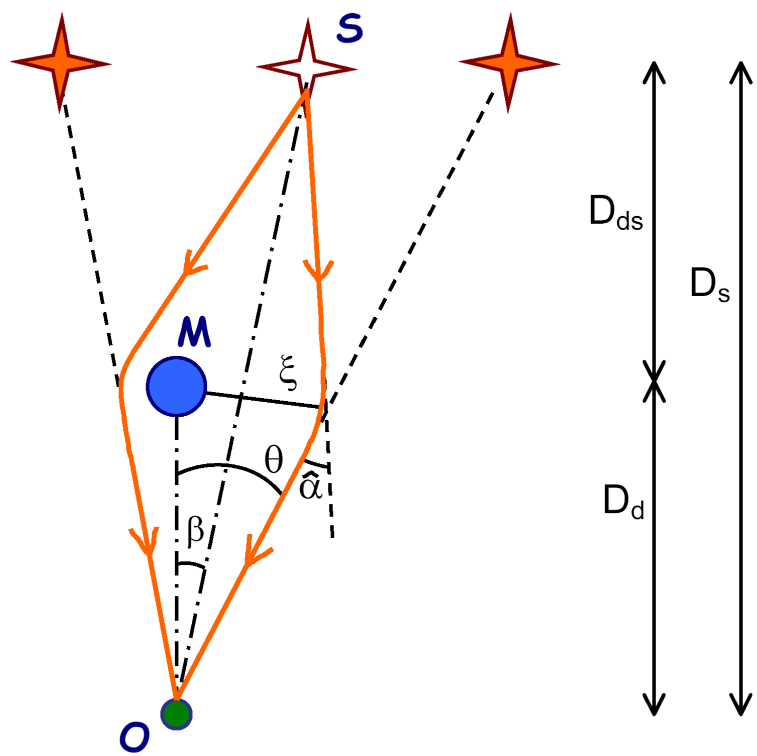
(a) If $b >> 3\sqrt{3}\,M$, then the orbit is almost a straight line with a small deflection by an angle $\hat{\alpha} = 4\,M/R$, where R is the distance of closest approach. This is the case customarily examined in the theory of weak gravitational lensing, when the impact parameter is much greater than the Schwarzschild radius of the lens.

(b) If $0 < b/M - 3\sqrt{3} << 1$, then the photon makes several turns around the black hole near a radius $r = 3M$ and flies off to infinity.

- Usual gravitational lensing is based on weak deflection limit:

$$\hat{\alpha} = 4\,M/R$$

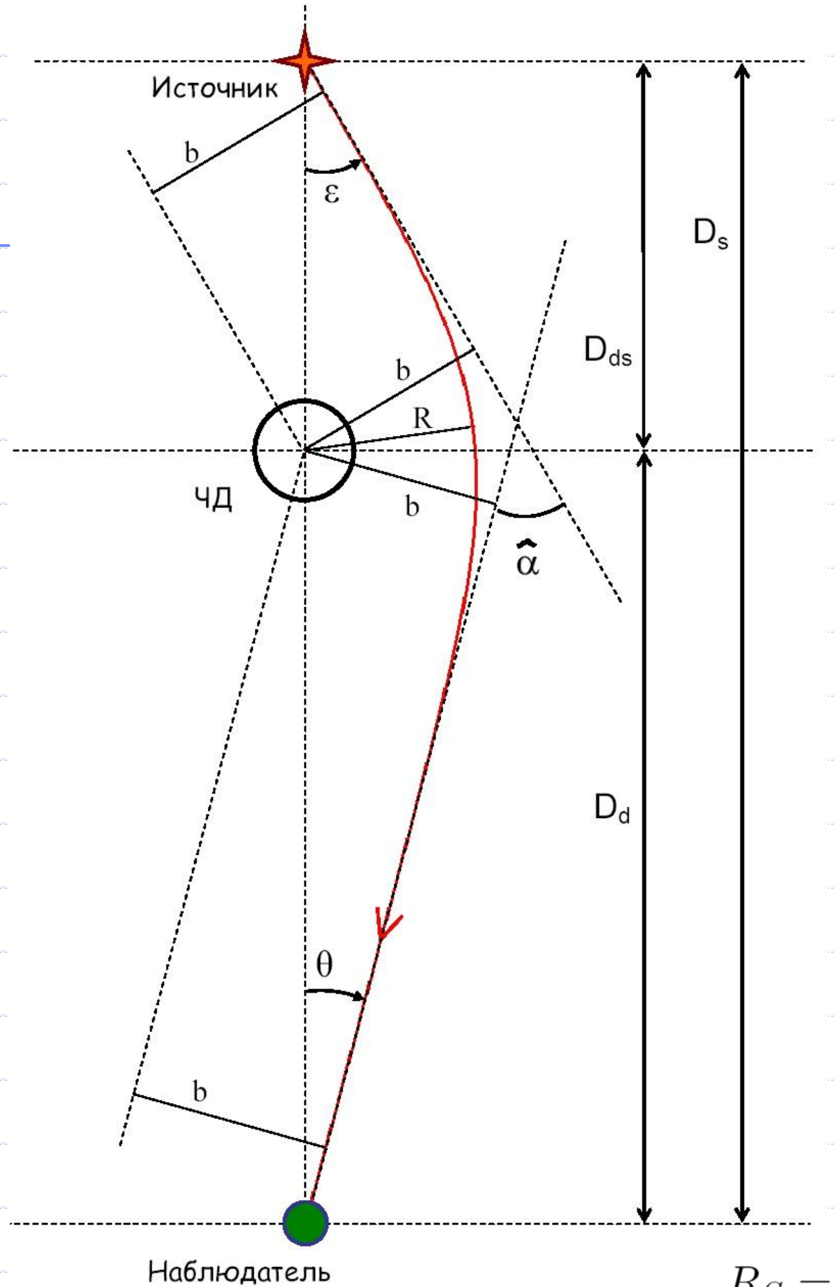
- The Schwarzchild lens



- In this work we investigate strong deflection of the photons near black hole and consider gravitational lensing by black hole in this situation

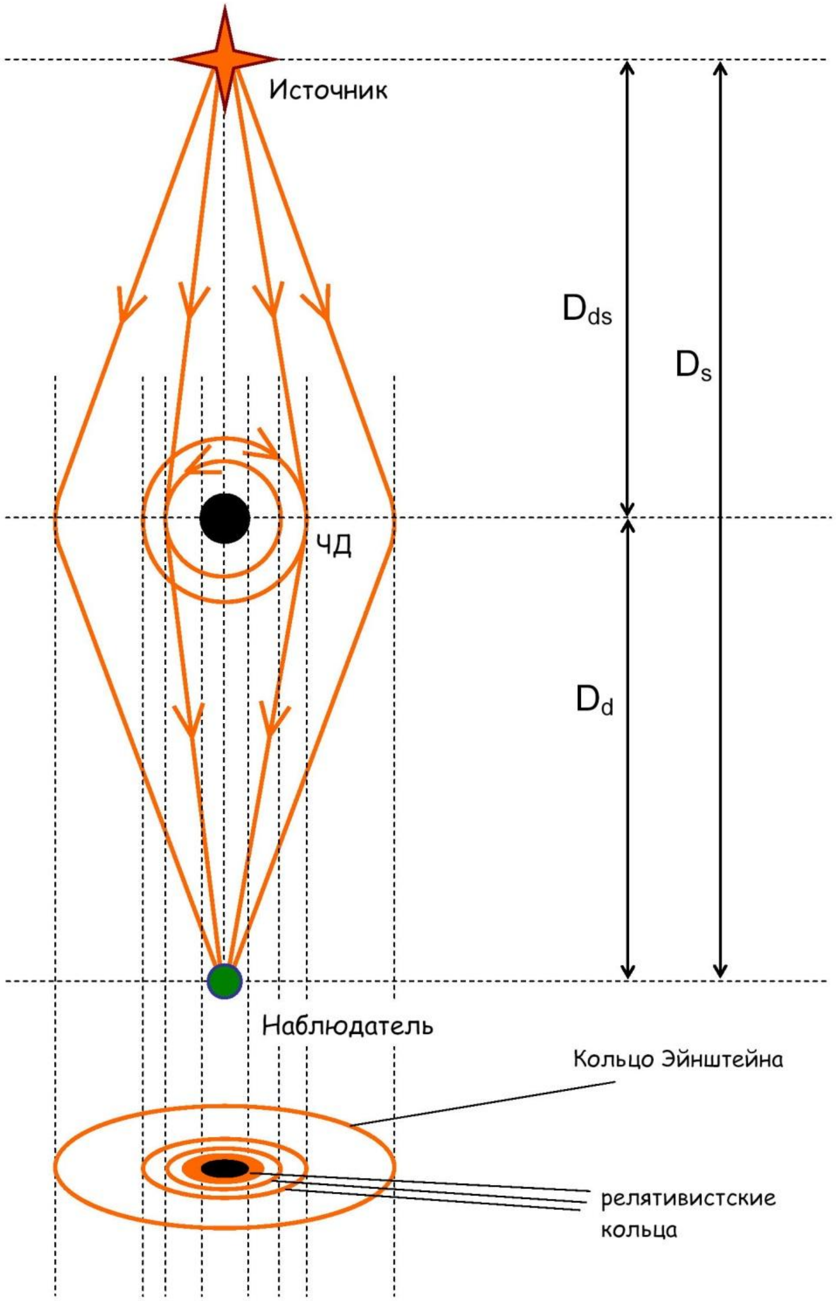
- Some works taking the motion of photons near the gravitational radius of a black hole into account:

- W. Ames and K. Thorne, *Astrophys. J*. 151, 659 (1968).
- G. S. Bisnovatyi-Kogan and A. A. Ruzmaikin, *Astrophys. Space Sci*. 28, 45 (1974).
- V. Bozza, S. Capozziello, G. Iovane, and G. Scarpetta, *General Relativity and Gravitation* 33, 1535 (2001).
- V. Bozza, *Phys. Rev. D* 66, 103001 (2002).
- C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, Freeman, New York (1973).
- C. Darwin, Proceedings of the Royal Society of London, Series A, *Mathematical and Physical Sciences* 249 (1257), 180 (1959).
- V. Bozza and M. Sereno, *Phys. Rev. D* 73, 103004 (2006).
- K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D 62, 084003, 2000.



- We will consider the case when the source, lens (black hole) and observer lie along a straight line

$$R_S = 2,\ R = 7,\ b \approx 8.3,\ D_d = 32,\ D_{ds} = 16.$$



- The angular radius of the Einstein ring

$$\theta_0 = \sqrt{4M\frac{D_{ds}}{D_d D_s}}$$

- From equations for orbit: $$\frac{d\varphi}{dr} = \frac{1}{r^2\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)}}$$

- Exact deflection angle for the photon:

$$\hat{\alpha} = 2\int_R^{\infty} \frac{dr}{r^2\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)}} - \pi$$

- R is the distance of the closest approach
- b is the impact parameter
- M is the mass of the black hole

$$\int_R^{\infty} \frac{dr}{r^2\sqrt{\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)}} = 2\sqrt{\frac{R}{Q}}\, F\left(\sqrt{\frac{8Q}{(6+Q-R)(R-2+Q)}}, \sqrt{\frac{6+Q-R}{2Q}}\right),$$

$$Q^2 = (R-2)(R+6)$$

$$F(z,k) = \int_0^z \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} = \int_0^{\arcsin\phi_0} \frac{d\phi}{\sqrt{(1-k^2\sin^2\phi)}}, \quad \sin\phi_0 = z\,.$$

The strong deflection limit

- If the value of the impact parameter is close to the critical value, we can also use approximate ***analytical*** formula for deflection angle (strong deflection limit). This formula is for case when photon incident from infinity undergoes one or more revolutions around the black hole and then escapes to infinity.

- Deflection angle as a function of distance of the closest approach

$$\hat{\alpha} = -2\ln\frac{R-3M}{36\left(2-\sqrt{3}\right)M} - \pi$$

- Relation between impact parameter and distance of the closest approach in case of strong deflection

$$b = 3\sqrt{3}\,M + \frac{\sqrt{3}}{2}\frac{(R-3M)^2}{M}$$

- Deflection angle as a function of distance the impact parameter

$$\hat{\alpha} = -\ln\left(b/M - 3\sqrt{3}\right) + \ln\left[648\left(7\sqrt{3}-12\right)\right] - \pi =$$

$$= -\ln\left(\frac{b}{b_{cr}} - 1\right) + \ln\left[216\left(7-4\sqrt{3}\right)\right] - \pi \simeq -\ln\left(\frac{b}{b_{cr}} - 1\right) - 0.40023$$

Positions of the relativistic rings

- Equating $\hat{\alpha}$ to $2\pi, 4\pi, 6\pi, \ldots,$ we find that relativistic rings are localized at the impact parameters

$$b/M - 3\sqrt{3} = 0.00653, 0.0000121, 0.0000000227, 0.423\cdot 10^{-10}, 0.791\cdot 10^{-13}\ldots$$

- The corresponding distances of closest approach are

$$R/M - 3 = 0.0902, 0.00375, 0.000162, 0.699\cdot 10^{-5}, 0.302\cdot 10^{-6}, \ldots$$

- The values of the impact parameters can also be obtained using the strong $\hat{\alpha}(b) = 2\pi n \quad (n = 1,\ 2,\ 3,\ \ldots)$ on

$$b_n = b_{cr}\left(1 + e^{C_1 - 2\pi n}\right), \quad C_1 = -0.40023.$$

- The angular sizes of the relativistic rings

$$\theta_n = \frac{b_{cr}}{D_d}\left(1 + e^{C_1 - 2\pi n}\right).$$

- D_d is the distance between lens and observer

Magnification factor for the main ring (Einstein ring)

We now find the magnification factor for the main image of a circular source with angular radius β and uniform brightness lying in a straight line with a lens and an observer. The image of the source is an annulus. We introduce $y = \beta/\theta_0$, the ratio of the angular size of the source to the angular size of the main ring. The unlensed source occupies a solid angle of $(\Delta\omega)_0 = \pi\beta^2 = \pi y^2 \theta_0^2$. It can be shown [1] that in the weak lensing limit, a circular source transforms into a ring with the following inner and outer angular radii: $\theta_{in} = \frac{\theta_0}{2}\left(\sqrt{y^2+4} - y\right)$ and $\theta_{out} = \frac{\theta_0}{2}\left(\sqrt{y^2+4} + y\right)$. Thus, the solid angle occupied by the ring is

$$\Delta\omega = \pi\left(\theta_{out}^2 - \theta_{in}^2\right) = \pi\frac{\theta_0^2}{4}\left[\left(\sqrt{y^2+4} + y\right)^2 - \left(\sqrt{y^2+4} - y\right)^2\right] = \pi\theta_0^2\, y\sqrt{y^2+4}\,. \qquad (22)$$

Thus, the magnification factor μ_0 for the main ring is

$$\mu_0 = \frac{\sqrt{y^2+4}}{y} \simeq \frac{2}{y}, \quad \text{for} \quad y \ll 1\,. \qquad (23)$$

Magnification factors for relativistic rings

We now find the flux "magnification" for the relativistic rings of this source. It can be shown, with using of Ref. 10, that the n-th relativistic ring will have the following inner and outer angular radii:

$$\theta_{in}^{n} = \theta_n (1 - A_n (\theta_n + \beta)), \quad \theta_{out}^{n} = \theta_n (1 - A_n (\theta_n - \beta)), \tag{24}$$

where

$$A_n = \frac{D_s}{D_{ds} D_d} \frac{b_{cr}}{\theta_n} e^{C_1 - 2\pi n} .$$

The solid angle occupied by this relativistic ring is

$$\Delta\omega = \pi \left[\left(\theta_{out}^{n}\right)^2 - \left(\theta_{in}^{n}\right)^2 \right] = 4\pi\theta_n^2 (1 - A_n \theta_n) A_n \beta . \tag{25}$$

Thus, the "magnification" factors μ_n of the relativistic rings are given by

$$\mu_n = \frac{4\theta_n^2 (1 - A_n \theta_n) A_n}{\beta} . \tag{26}$$

The term

$$A_n\,\theta_n = \frac{D_s}{D_{ds}D_d}\, b_{cr}\, e^{C_1-2\pi n} << 1, \tag{27}$$

so that, to great accuracy, we have

$$\mu_n = 4\frac{\theta_n}{\beta} A_n\,\theta_n\,. \tag{28}$$

After some transformations, we obtain

$$\mu_n = 4\frac{b_{cr}^2}{\beta}\frac{D_s}{D_{ds}D_d^2}\left(1+e^{C_1-2\pi n}\right)e^{C_1-2\pi n} << \mu_0\,. \tag{29}$$

Using the expressions for $b_{cr} = 3\sqrt{3}\,M = \left(3\sqrt{3}/2\right)R_S$ and $\beta = R_*/D_s$, where R_* is the source radius, we obtain

$$\mu_n = 27\frac{R_S^2 D_s^2}{R_* D_{ds} D_d^2}\left(1+e^{C_1-2\pi n}\right)e^{C_1-2\pi n} << 1. \tag{30}$$

For a distant quasar with $M_* = 10^9\,M_\odot$, $R_* = 15\,R_{*S}\left(R_{*S} = 2\,GM_*/c^2\right)$, $D_{ds} = 10^3$ Mpc, D_d=3 Mpc, $D_s \approx D_{ds}$, and a lens of mass $M = 10^7\,M_\odot$, we obtain

$$\mu_0 = 1\cdot 10^6,\quad \mu_1 = 2\cdot 10^{-15},\quad \mu_2 = 4\cdot 10^{-18}. \tag{31}$$

For the same quasar with a lens of mass $M = 20\,M_\odot$ and D_d=1 kpc, we obtain

$$\mu_0 = 9\cdot 10^4,\quad \mu_1 = 9\cdot 10^{-20},\quad \mu_2 = 2\cdot 10^{-22}. \tag{32}$$

4. Гравитационное линзирование в плазме, *гравитационный радиоспектрометр*

G.S. Bisnovatyi-Kogan, O.Yu. Tsupko

Gravitational Radiospectrometer, *Gravitation and Cosmology, accepted.*
arXiv:0809.1021v2 [astro-ph] 19 Sep 2008

- An ordinary theory of the gravitational lensing is developed for the light propagation in the vacuum. Deflection angle does not depend on frequency of the photon and determined only by the impact parameter. So gravitational lensing is vacuum is achromatic.

- It is well known that in inhomogeneous medium photons moves along curved trajectory, and if medium is dispersive the trajectory depends on frequency of the photon.

In medium the light rays move with the group velocity. For plasma with the index of refraction $n^2 = 1 - \omega_e^2/\omega^2$ the group velocity is $v_{gr} = cn$. The smaller frequency and bigger wavelength correspond to the smaller group velocity.

Gravitational lensing in plasma, previous results

- D. O. Muhleman and I. D. Johnston, Phys. Rev. Lett. 17, 8, 455 (1966).
- D. O. Muhleman, R. D. Ekers, and E. B. Fomalont, Phys. Rev. Lett., 24, 24, 1377 (1970).
- P. V. Bliokh and A. A. Minakov, Gravitational Lenses (Naukova Dumka, Kiev, 1989), in Russian.

In previous papers of different authors concerning deflection of light by both gravitation and plasma there was separated consideration of two effects:

Gravitational deflection of light in vacuum

It does not depend on frequency

+

Deflection of light in non-homogeneous medium (non-relativistic effect)

It depends on frequency if the medium is dispersive, but is equal to zero if the medium is homogeneous

The new result:

In this work we show that due to dispersive properties of plasma even in the homogeneous plasma gravitational deflection will differ from vacuum deflection angle, and gravitational deflection angle in plasma will depend on frequency of the photon.

Self-consistent approach for geometrical optics in curved space-time in medium:
J.L. Synge, Relativity: the General Theory, North-Holland Publishing Company, Amsterdam, 1960.

On basis of his general approach we developed the model of gravitational lensing in plasma.

We consider a static space-time with the metric

$$ds^2 = g_{ik}\, dx^i dx^k = g_{\alpha\beta}\, dx^\alpha dx^\beta + g_{00}\left(dx^0\right)^2, \quad i,k = 0,1,2,3, \quad \alpha,\beta = 1,2,3. \tag{1}$$

Here g_{ik} do not depend on the time. Gravitational field is week:

$$g_{ik} = \eta_{ik} + h_{ik}, \quad h_{ik} \ll 1, \quad h_{ik} \to 0 \ \text{ under } \ x^\alpha \to \infty\,. \tag{2}$$

Here η_{ik} is the metric of a flat space, and h_{ik} is a small perturbation.

Let us consider in this gravitational field a static inhomogeneous plasma with the refraction index n, which depends on the space location x^α and the frequency of the photon $\omega(x^\alpha)$:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(x^\alpha)]^2}, \quad \omega_e^2 = \frac{4\pi e^2 N(x^\alpha)}{m}. \tag{3}$$

Here e is the charge of the electron, m is the mass of the electron, $N(x^\alpha)$ is the electron concentration in the inhomogeneous plasma, ω_e is the electron plasma frequency in this plasma.

$$N(x^\alpha) = N_0 + N_1(x^\alpha), \quad N_0 = \text{const}, \quad N_1(\infty) = 0. \tag{4}$$

$$\omega_e^2 = \omega_0^2 + \omega_1^2, \ \text{ where } \ \omega_0^2 = K_e N_0, \quad \omega_1^2 = K_e N_1, \quad K_e = \frac{4\pi e^2}{m}. \tag{5}$$

If the problem is axially symmetric, it is convenient to introduce the impact parameter b, and we obtain for the deflection angle of the photon moving along z-axis

$$\hat{\alpha}_b = \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{\partial h_{33}}{\partial b} + \frac{\omega^2}{\omega^2 - \omega_0^2} \frac{\partial h_{00}}{\partial b} - \frac{K_e}{\omega^2 - \omega_0^2} \frac{\partial N_1(r)}{\partial b} \right) dz$$

Here e is the charge of the electron, m is the mass of the electron, $N(x^\alpha)$ is the electron concentration in the inhomogeneous plasma, ω_e is the electron plasma frequency in this plasma.

$$N(x^\alpha) = N_0 + N_1(x^\alpha), \quad N_0 = \text{const}, \quad N_1(\infty) = 0. \tag{4}$$

$$\omega_e^2 = \omega_0^2 + \omega_1^2, \quad \text{where} \quad \omega_0^2 = K_e N_0, \quad \omega_1^2 = K_e N_1, \quad K_e = \frac{4\pi e^2}{m}. \tag{5}$$

When gravitating body is surrounded by a plasma, the lensing angle depends on a frequency of the electromagnetic wave due to refraction properties, and the dispersion properties of the light propagation in plasma.

Gravitational radiospectrometer

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

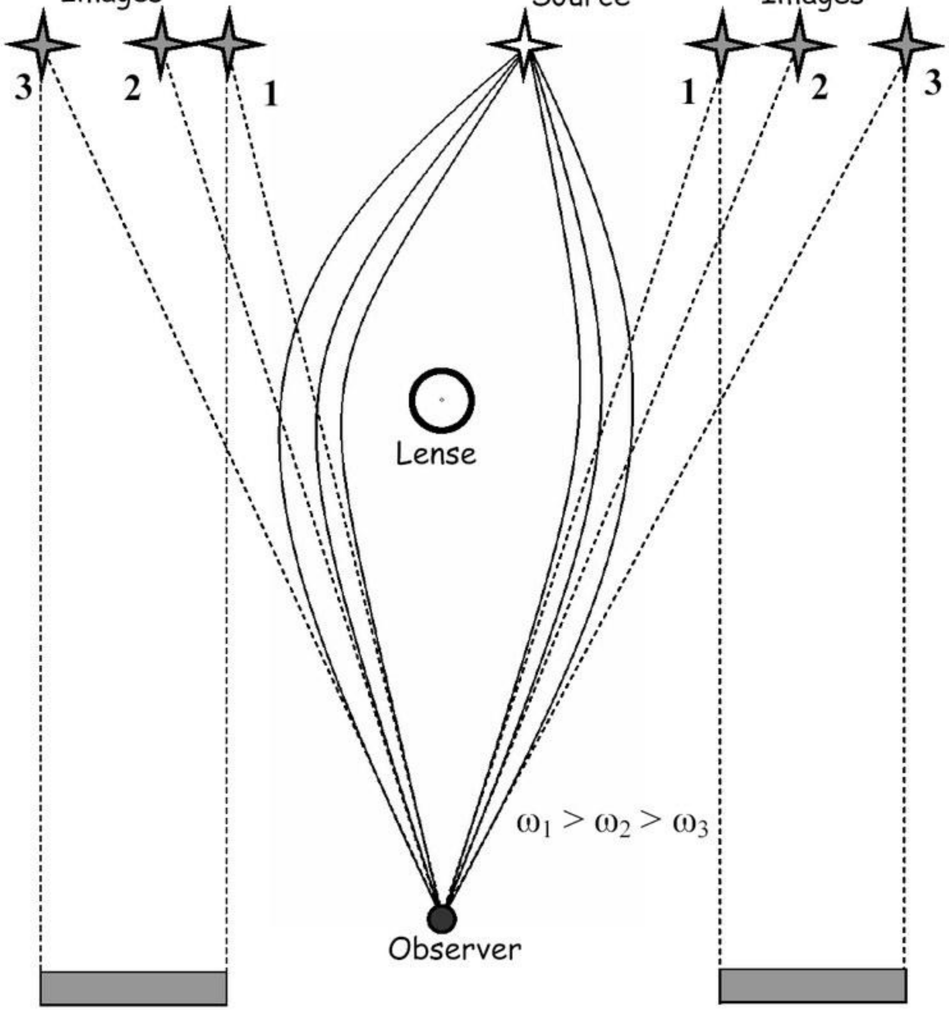
in vacuum
(N_1=N_0=0)

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - \omega_0^2/\omega^2}\right)$$

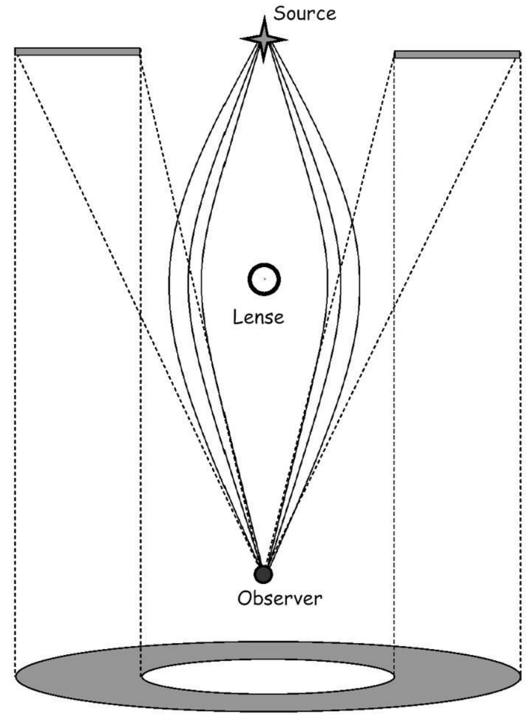
in homogeneous plasma (N_1=0)

When gravitating body is surrounded by a plasma, the lensing angle depends on a frequency of the electromagnetic wave due to refraction properties, and the dispersion properties of the light propagation in plasma. The last effect leads to dependence, even in the homogeneous plasma, of the lensing angle on the frequency, what resembles the properties of the refractive prism spectrometer. The strongest action of this spectrometer is for the frequencies slightly exceeding the plasma frequency, what corresponds to very long radiowaves.

GRAVITATIONAL RADIOSPECTROMETER



Instead of two concentrated images with complicated spectra, we will have two line images, formed by the photons with different frequencies, which are deflected by different angles.



$\lambda_1 < \lambda_2 < \lambda_3$

$v_{gr1} > v_{gr2} > v_{gr3}$

$\alpha_1 < \alpha_2 < \alpha_3$

The smaller group velocity (smaller frequency and bigger wavelength) corresponds to a larger deflection angle.

Observations in vacuum

The Einstein angle of the photon deflection in vacuum is

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b},$$

under condition $b \gg R_S$, where b is the impact parameter, R_S is the Schwarzschild gravitational radius of body with mass M.

Typical angular separation between images is:

The radius of Einstein ring

$$\alpha_0 = \sqrt{2R_S \frac{D_{ds}}{D_d D_s}},$$

where D_d is the distance between observer and lens, D_s is the distance between observer and source, D_{ds} is the distance between lens and source.

Typical angular separation for quasars is about 1 arcsec.

Observations in homogeneous plasma

Deflection angle in a homogeneous plasma is:

$$\hat{\alpha} = \frac{R_S}{b} \left(1 + \frac{1}{1 - \omega_0^2/\omega^2} \right),$$

where $\omega_0^2 = \frac{4\pi e^2}{m} N_e$ is the electron plasma frequency, N_e is the electron concentration in homogeneous plasma, m is the mass of the electron, ω is the frequency of the photon (at infinity, $\sec^{-1}$).

Typical angular separation between images in homogeneous plasma is:

The radius of "plasma" Einstein ring

$$\alpha_0 = \sqrt{\left(1 + \frac{1}{1 - \omega_0^2/\omega^2} \right) R_S \frac{D_{ds}}{D_d D_s}}$$

Difference in the typical angular separation of images (between vacuum and plasma) is:

$$\frac{\delta\alpha_0}{\alpha_0} \simeq 2 \cdot 10^7 \, \frac{N_e}{\nu^2},$$

where ν is the photon frequency (Hz), $\omega = 2\pi\nu$.

Observations at **RadioAstron**

http://www.radioastron.ru/

Difference in the typical angular separation of images (between vacuum and plasma) is:

$$\frac{\delta\alpha_0}{\alpha_0} \simeq 2 \cdot 10^7 \, \frac{N_e}{\nu^2},$$

where ν is the photon frequency (Hz), $\omega = 2\pi\nu$.

Frequency band (GHz) : 0.327 (P);
1.665 (L);
4.830 (C);
18.392-25.112 (K)

For the lowest frequency of RadioAstron $\nu = 327 \cdot 10^6$ Hz,

Angle difference between vacuum case and plasma case 0.00001 arcsec will be at N_e ~ 50 000 cm^{-3}

Сравнение с работами других авторов

- Kulsrud, Loeb, 1992, PhRvD..45..525K
- Broderick, Blandford, 2003 Ap&SS.288..161B
- Broderick, Blandford, 2003 MNRAS.342.1280B

In homogeneous plasma with refraction index $n^2 = 1 - \omega_0^2/\omega^2$ photon moves like a massive particle of mass ω_0, energy ω and velocity equal to group velocity of the photon.

Deflection angle for massive particle ($G = c = 1$):

$$\alpha = \frac{2M}{b}\left(1 + \frac{1}{\beta^2}\right)$$

Substituting $\beta \to v_{gr} = n$, we obtain

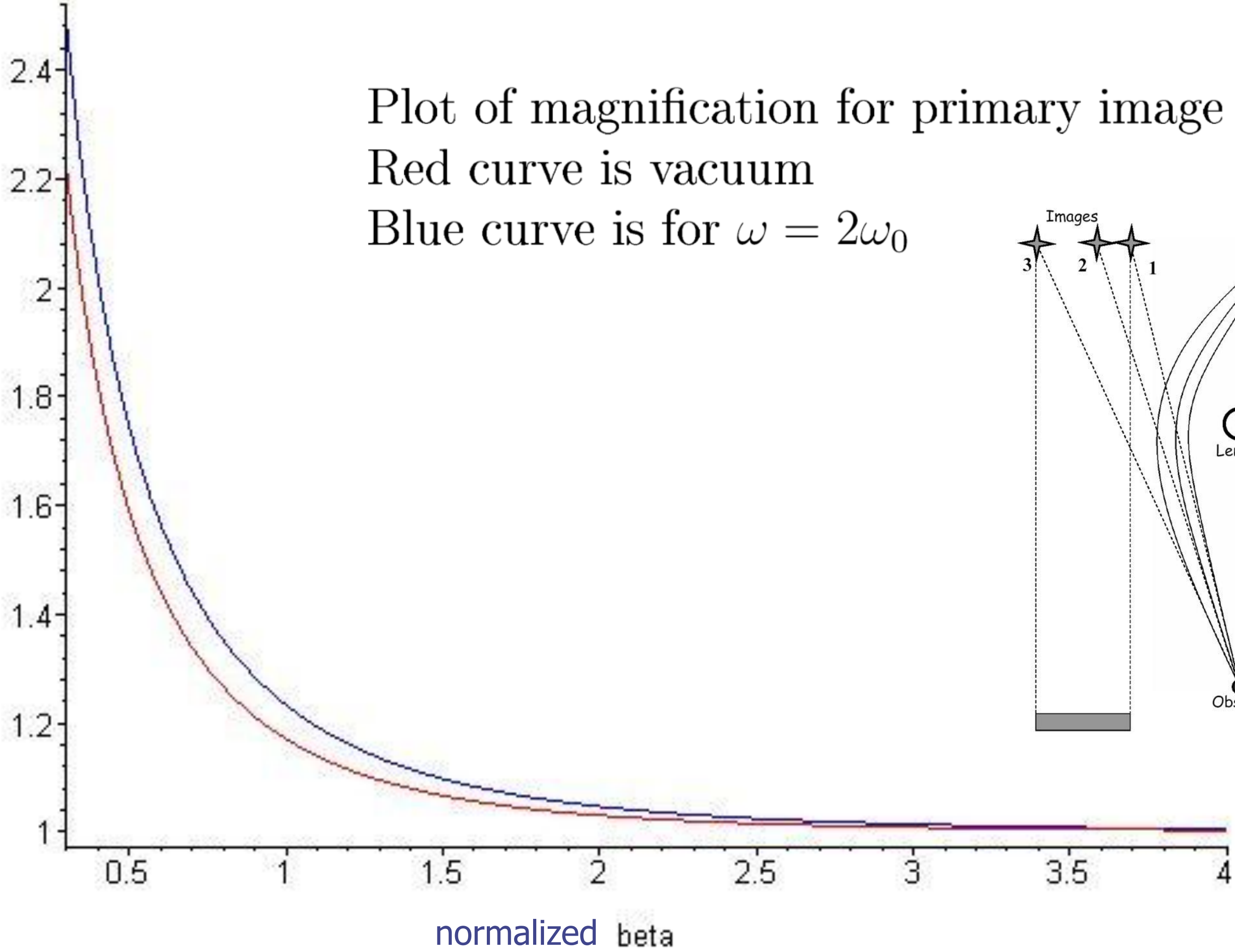
$$\hat{\alpha} = \frac{2M}{b}\left(1 + \frac{1}{1 - \omega_0^2/\omega^2}\right)$$

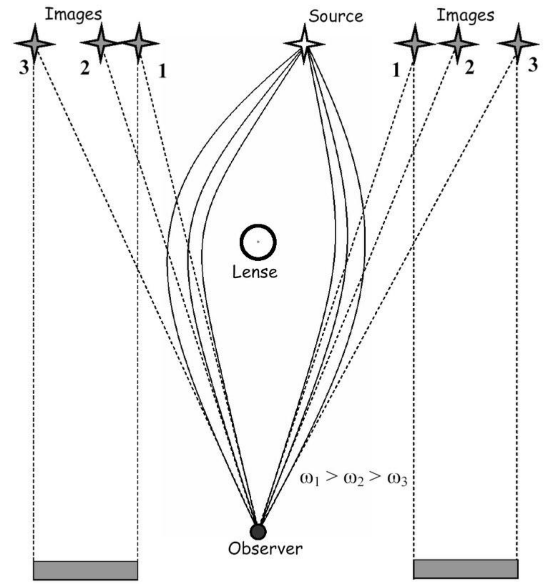
Magnification of the image depends on the lensing angle, therefore different images may have different spectra in the radio band, when the light propagates in regions with different plasma density.

For the Schwarzschild lense the magnification is proportional to the angular radius of the "plasma" Einstein ring, which increases when the frequency approaches the plasma frequency.

$$\alpha_0 = \sqrt{\left(1 + \frac{1}{1 - \omega_0^2/\omega^2}\right) R_S \frac{D_{ds}}{D_d D_s}}$$

Plot of magnification for primary image
Red curve is vacuum
Blue curve is for $\omega = 2\omega_0$





Normalized beta = beta/alpha_0(vacuum)

beta is angular position of the source
Point source transforms to two images.

Conclusions

Gravitational lens in plasma acts as a *gravitational radiospectrometer*

Expectations for the observations:

1) Extended image may have different spectra along the image

2) Spectra of point source images may be different in the long wave side

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